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Reply

R. L. Naff

U.S. Geological Survey, Denver, Colorado

T.-C. Jim Yeh

Department of Hydrology and Water Resources, University of Arizona, Tucson

M. W. Kemblowski

Shell Development Company, Houston, Texas

We are pleased to receive this very illuminating comment as to the inspiration behind the two-dimensional model used to fit the Borden site natural gradient tracer test data. At the time we conceived our original note we were aware that the field transverse moments were not in agreement with the theoretical transverse variance of the three-dimensional model; however, we considered this disagreement to be a problem with the field test, not the three-dimensional model. Our opinion in this regard has not really changed; we will pursue in more detail our doubts that the field experiment complies with flow criterion set forth by these models.

To restate the problem, as we see it, the theoretical variances for three-dimensional steady flow in heterogeneous porous media do not capture certain observed elements of the Borden site natural gradient test; this departure from three-dimensional flow theory is most evident because of the large degree of stratification found at this site. In particular, the model horizontal transverse variance σ_{22} is not of the magnitude of that actually observed, and the relationship of this moment with the vertical transverse moment σ_{33}, as predicted by theory ($\sigma_{33} > \sigma_{22}$), is contrary to field observation. From our own ongoing work we can confirm that these theoretical considerations are basically correct and that local dispersion will not significantly alter their relative magnitudes. Thus we are also in agreement with *Dagan*'s [this issue] conclusion that the existing moment model for three-dimensional steady flow does not accurately reflect the observed transverse moments; indeed, not only are the Borden site results afflicted in this manner but so are the Cape Cod site results [*Garabedian*, 1987], where a similar natural gradient tracer test has been performed in a somewhat similar, stratified aquifer. There is, however, a subtle difference between the Borden site and Cape Cod site results: while at both sites the estimated moments $\tilde{\sigma}_{j\ell}$ have the same relative relationship, $\tilde{\sigma}_{11} > \tilde{\sigma}_{22} > \tilde{\sigma}_{33}$, at the Cape Cod site the $\tilde{\sigma}_{22}$ moment, with respect to time, exhibits a markedly linear behavior ([*Garabedian*, 1987, Figure 68]; a horizontal transverse dispersivity of about 0.02 m is estimated). The time-dependent behavior of $\tilde{\sigma}_{22}$ at the Borden site is rather nebulous, allowing the large-time logarithmic behavior of the theoretical transverse two-dimensional moment [*Dagan*, 1984, equation (4.7)] to be fit to the estimated transverse moment. Because of this difference in time-dependent behavior, we suspect that the two-dimensional moment model will not be of any great utility at the Cape Cod site; this doubt gives us cause to suspect the universality of the two-dimensional model. In our opinion, if the moment models for three-dimensional steady flow in heterogeneous porous media are basically correct, as we suspect they are, then these moment models are telling us that the flow fields at these sites, in some material way, are significantly different from the model. Indeed, that such small transverse moments for a stratified medium are predicted by theory [*Dagan*, 1988] causes us to wonder whether some other agent is not responsible for the observed transverse field moments; this agent could be masking the transverse variability in the flow field by a slightly more significant variability of its own.

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Our hypothetical agent of favor to explain this discrepancy is essentially that of *Sudicky* [1986, p. 2080]; that is, small-scale transients have so overwhelmed the variation due to heterogeneity that this latter variation cannot be detected. *Rehfeldt* [1988] has expanded on this hypothesis to develop theoretical macrodispersivities when the flow field is a stationary random function of time and space. That is, he assumes that the presence of small-scale transients causes variations in the flow field which are negligible when averaged over large time, giving the flow field the appearance of being in steady state. In terms of a Lagrangian formulation, this assumption would require that an additional time dependency be included in the Eulerian description of the flow field, $\mathbf{U}(\mathbf{x}, t)$ [see *Naff et al.*, 1988, equation (1)]:

$$\boldsymbol{\chi}(t) = \int_0^t \mathbf{U}(\boldsymbol{\chi}(t), t)\, dt \tag{1}$$

where $\boldsymbol{\chi}(t)$ is the Lagrangian position vector. *Rehfeldt* [1988] assumes that this additional dependency is manifested in the gradient such that, while the mean of the gradient is constant, the perturbation about the mean contains a time and space dependency. While Rehfeldt's development is interesting, it does not readily lend itself well to a moment analysis, as it considers only large-time effects.

To facilitate an understanding of the possible effects of small-scale transients in the moments we can perform a crude evaluation of these effects by assuming that spatial and

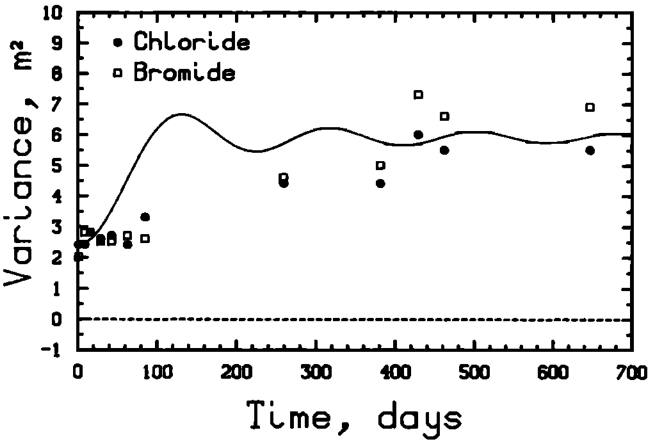

Fig. 1. Fit of transverse moment data from Borden site with one harmonic.

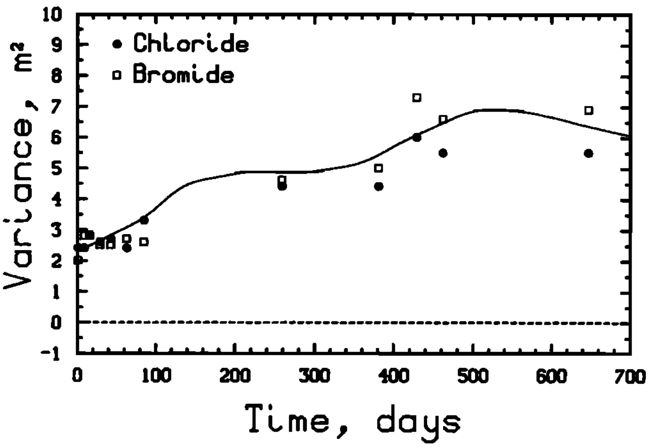

Fig. 2. Fit of transverse moment data from Borden site with three harmonics.

temporal variations in the flow field are essentially independent and that the temporal variation of the ith component of the velocity field, U_i, can be modeled with the following deterministic expression:

$$U_i(\mathbf{x}, t) = K_i[\delta_{1i}J + a_i \cos(2\pi \bar{U}_1 t/\ell)]/n \tag{2}$$

where ℓ is the travel distance associated with the mean velocity $\bar{\mathbf{U}} = (\bar{U}_1, 0, 0)$, $\bar{U}_1 = K_1 J/n$, J is the mean gradient in the x_1 direction, K_i is the effective hydraulic conductivity in the ith direction, and a_i is a measure of the temporal variability of the gradient in this direction. We adopt this simple, deterministic approximation of the transient behavior because the true velocity field, $\mathbf{U}(\mathbf{x}, t)$, will be a rather complex function of the recharge process, which itself is a function of space and time. The variation in particle position for the ith direction, χ_i', about its mean, $\bar{\chi}_i = \delta_{1i}\bar{U}_1 t$, becomes

$$\chi_i'(t) = \Gamma_i \bar{U}_1 G_i \int_0^t \cos(2\pi \bar{U}_1 t/\ell)\, dt \tag{3}$$

where $G_i = a_i/J$ is a measure of the relative temporal variability in the gradient, and $\Gamma_i = JK_i/\bar{U}_1 n$. As a measure of the variance $\hat{\sigma}_{ii}$ in particle position in the ith direction we use the following approximate average:

$$\hat{\sigma}_{ii} = t^{-1} \int_0^t [\chi_i'(t)]^2\, dt \tag{4}$$

Integrating (4) with (3) produces the following simplistic model of the ith second moment:

$$\hat{\sigma}_{ii} = \ell^2 \Gamma_i^2 G_i^2 \left[1 - \frac{1}{4\pi\tau} \sin(4\pi\tau)\right] / 8\pi^2 \tag{5}$$

where $\tau = t\bar{U}_1/\ell$. When G_i, $i = 2$, is given a value of 0.5 and ℓ is assumed to be a length equivalent to a travel time of 1 year, then the estimated transverse moments of *Freyberg* [1986] can be modeled as shown in Figure 1 (Γ_2 becomes unity if $K_1 = K_2$, as assumed here). While interesting, this model is obviously inadequate as it places all the temporal variation on one harmonic. If, for instance, it is assumed that three harmonics are necessary to account for the temporal variability such that

$$\chi_i'(t) = \Gamma_i \bar{U}_1 \int_0^t \sum_{m=1}^{3} G_{im} \cos(2\pi \bar{U}_1 t/\ell_m)\, dt \tag{6}$$

where now G_{im} is the relative variability in gradient associated with the mth harmonic and ℓ_m is the attendant travel length, a more realistic model can be obtained by substituting this expression into (4) and integrating. By letting ℓ_1 be a travel length equivalent to $\frac{1}{4}$ year, ℓ_2 be equivalent to 1 year, and ℓ_3 be equivalent to 4 years, and letting G_{2m}, $m = 1, 2, 3$, take on the values 0.1, 0.14, and 0.12, respectively, the fit shown in Figure 2 can be obtained. The fit now is quite good; however, the whole procedure smacks of curve fitting. What is more important is that these relative variabilities G_{2m} of the gradient are rather smaller than in the single harmonic case, indicating that small temporal variations of this nature could explain the anomalous transverse moments at both the Borden and Cape Cod sites.

Obviously, to verify that transients were responsible for the increase transverse moments at both the Borden and Cape Cod sites, one would need to collect an extensive time series of water level fluctuations from closely spaced piezometers from these sites and do a harmonic analysis on the estimated gradients. *Mackay et al.* [1986], citing *Macfarlane et al.* [1983], note the presence of significant variations in the gradient at the Borden site. *Rehfeldt* [1988] has shown that, at yet another site, gradient variations are observable and can be measured. With regard to the Borden site, using a few approximations, *Rehfeldt* [1988] found that the large-time transverse horizontal macrodispersivity can be estimated to within an order of magnitude of the actual value. The concept of small-scale transients affecting transverse dispersion is not new: *Ackerer and Kinzelbach* [1985] postulated such an effect to explain excessive transverse spreading of a plume in Germany. Note that models such as (4) and (6) indicate that the longitudinal moment as well will be affected by transients; however, as these effects in the longitudinal should be of the same order of magnitude as the transverse, their contribution to the longitudinal moment, as compared to that of heterogeneity, should be small.

In conclusion, we quite agree with *Dagan* [this issue] concerning the suggestion that additional field experiments or theoretical developments are necessary in order to draw more definite conclusions about the accuracy of these models; however, his conjecture concerning the validity of the two-dimensional model to represent the plume at the Borden site is, for us at least, unconvincing.

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T.-C. Jim Yeh, Department of Hydrology and Water, University of Arizona, Tucson, AZ 85721.

M. W. Kemblowski, Shell Development Company, P. O. Box 1380, Houston, TX 77251.

R. L. Naff, U.S. Geological Survey, Box 25046, MS 413, Denver, CO 80225.

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