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Comment on "The Role of Groundwater in Delaying Lake Acidification" by M. P. Anderson and C. J. Bowser

ROBERT W. RITZI, JR. AND T.-C. JIM YEH

Department of Hydrology and Water Resources, University of Arizona, Tucson

Anderson and Bowser [1986], hereafter referred to as AB, presented a very interesting paper on the study of the role of groundwater in delaying lake acidification in Wisconsin. By considering the input/output of hydrogen ion with respect to the aquifer, including nonconservative transport through the use of the retardation factor, the effect of a delay in the arrival of acid to the lake is demonstrated. A two-dimensional, finite difference flow and random walk transport model, neglecting dispersion, was used to simulate the migration of the acid water through the groundwater system. Also, the temporal changes in lake pH were examined by modeling the lake with both a lumped parameter (well-mixed) model and an equilibrium chemistry model. Thus the lake model mixes inflow from the groundwater to the lake, precipitation on lake surface, and lake water, with the consideration of equilibrium chemical reactions. From their work they concluded that groundwater inflow can mitigate the effect of acid deposition on lake watersheds, as well as the fact that the low ionic strength of the lakes was an important buffer. The scope of this comment is confined only to the issue of modeling the buffering capability of the aquifer. The purpose is to show that an alternate and simpler formulation for the groundwater model is also appropriate for the case study of AB.

Lumped parameter models for surface water systems have been widely used by engineers in the past few decades. *Gelhar and Wilson* [1974], hereafter referred to as GW, developed a lumped parameter model for groundwater systems to investigate the effect of deicing salt on groundwater quality. Since then the lumped parameter approach has been applied to only few groundwater related problems [e.g., *Updegraff and Gelhar*, 1977; *Simonent*, 1981]. Generally, the usefulness of this approach has not been fully recognized in the field of groundwater hydrology. In this comment we demonstrate the utility of a lumped parameter approach for the aquifer-lake system in Wisconsin examined by AB. We also hope, through the example, to promote the use of such an approach for other appropriate groundwater quality problems.

As is pointed out by GW, a lumped parameter approach will be justified for cases of distributed contaminant sources, for cases of limited data on spatial variation of aquifer properties, or for cases in which temporal variation in mean contaminant concentration is of primary importance. In the study by AB, acid deposition is assumed a uniformly distributed contaminant source in space. Only average output concentrations of hydrogen ion entering the lake are of concern while spatial variations within the groundwater system itself are ignored. Thus a lumped parameter approach is appropriate for this study.

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In the following analysis, the groundwater quality model for the groundwater component of the system in Wisconsin is reformulated by a lumped parameter approach. Because AB only present their results in terms of lake concentrations, the two groundwater models cannot be directly compared. Thus the lumped parameter model is linked with a well-mixed lake model to compare the resulting water quality of the lake to the results of AB. Note that the scope of this analysis concerns only the formulation of the groundwater model. The respective groundwater modeling methodologies can be compared without reperforming the equilibrium chemistry analysis in the lake, and thus that component of the previous work (by AB) is not considered here.

Following the mass balance of GW, the groundwater balance equation is expressed as

$$n\frac{dh}{dt} = \varepsilon - q \tag{1}$$

where n is the porosity, h is the average thickness of the saturated zone, ε is the recharge rate per unit area of the aquifer, and q is discharge from the aquifer. Using a linear reservoir assumption, q can be approximated as

$$q = a(h - h(0)) \quad a = \frac{3T}{L^2}$$

where T is the transmissivity, L is the length of the aquifer, and $h(0)$ is the initial average thickness of the saturated zone. For a steady state flow, h can be approximated by $h(0) + \varepsilon/a$.

The chemical balance for the aquifer system can be formulated as

$$n\frac{dhC_a}{dt} = C_\varepsilon\varepsilon - C_a q - Sh \tag{2}$$

$$S = (1-n)\rho_s Kd\frac{dC_a}{dt} \tag{3}$$

where ρ_s is the density of solid, Kd is the distribution coefficient, C_a is the average concentration of the aquifer, and C_ε is the concentration in recharge.

Combining (1), (2), and (3), one can obtain a chemical balance equation for the aquifer system:

$$\frac{dC_a}{dt} + AC_a = AC_\varepsilon \tag{4}$$

where $A = \varepsilon/nhR$ and the retardation factor $R = (1 + (1-n)/n\rho_s Kd)$. The solution to (4) for a step input concentration is given

$$C_a(t) = C_a(0)\exp(-At) + C_\varepsilon(1 - \exp(-At)) \tag{5}$$

where $C_a(0)$ is the initial aquifer concentration.

Similarly, one can formulate the chemical mass balance for

TABLE 1. Values of the Parameters Used in the Lumped Parameter Model

Parameter	Value
ε	2.5×10^{-3} ft/day
n	0.3
T	3,000 ft^2/day
L	8,350 ft
$h(0)$	100 ft
h	119 ft
$C_a(0)$	$10^{-5.8}$
C_ε	$10^{-4.6}$
C_p	$10^{-4.6}$
$C_\ell(0)$	$10^{-5.8}$
q	17 ft^2/day
p	21.2 ft^2/day
V	104,500 ft^2

One foot (ft) equals 30.48 cm; 1 ft^2 = 0.093 m^2.

the lake system as

$$\frac{dC_l}{dt} + BC_l = I \tag{6}$$

where C_l is the concentration in the lake; $B = (p + q)/V$; $I = (C_p p + C_a q)/V$; V is the volume of the lake; C_p is the concentration in precipitation, p is the precipitation rate per unit area; and C_a is the concentration in the aquifer outflow (5). The kernel for (6) is simply an exponential decay, and so the solution to (6) with time-dependent groundwater concentration (5) as well as acid rain as input can be obtained by the convolution

$$C_l(t) = \int_0^t I(\tau) \exp(-B(t-\tau))d\tau$$

giving the solution for lake concentration in time as

$$C_l(t) = E[1 - \exp(-Bt)] + D[\exp(-At) - \exp(-Bt)] + C_l(0)\exp(-Bt) \tag{7}$$

where $E = C_p p + C_\varepsilon q/VB$; $D = q(C_a(0) - C_\varepsilon)/[V(B - A)]$; and $C_l(0)$ is the initial lake concentration. To demonstrate the applicability of model (7) to the aquifer-lake system in Wisconsin, a steady state flow is assumed, and the parameters are given numerical values based on system 1 defined in the paper by AB. These values are summarized in Table 1. Those values which are not explicitly stated by *Anderson and Bowser* [1986] were obtained from them by personal communication.

An analytical solution such as (7) will always have one advantage over a numerical solution (AB) in that the system can be understood without performing a large number of simulations, as is necessary in the numerical case. To illustrate this advantage of the lumped parameter model, a first-order sensitivity analysis [*Cornell*, 1972] of the model was performed to study the effect of aquifer parameters on the lake response.

The first-order sensitivity analysis allows for a consideration of the variation in the model output $g(\theta_1, \theta_2, \cdots, \theta_n)$, due to a

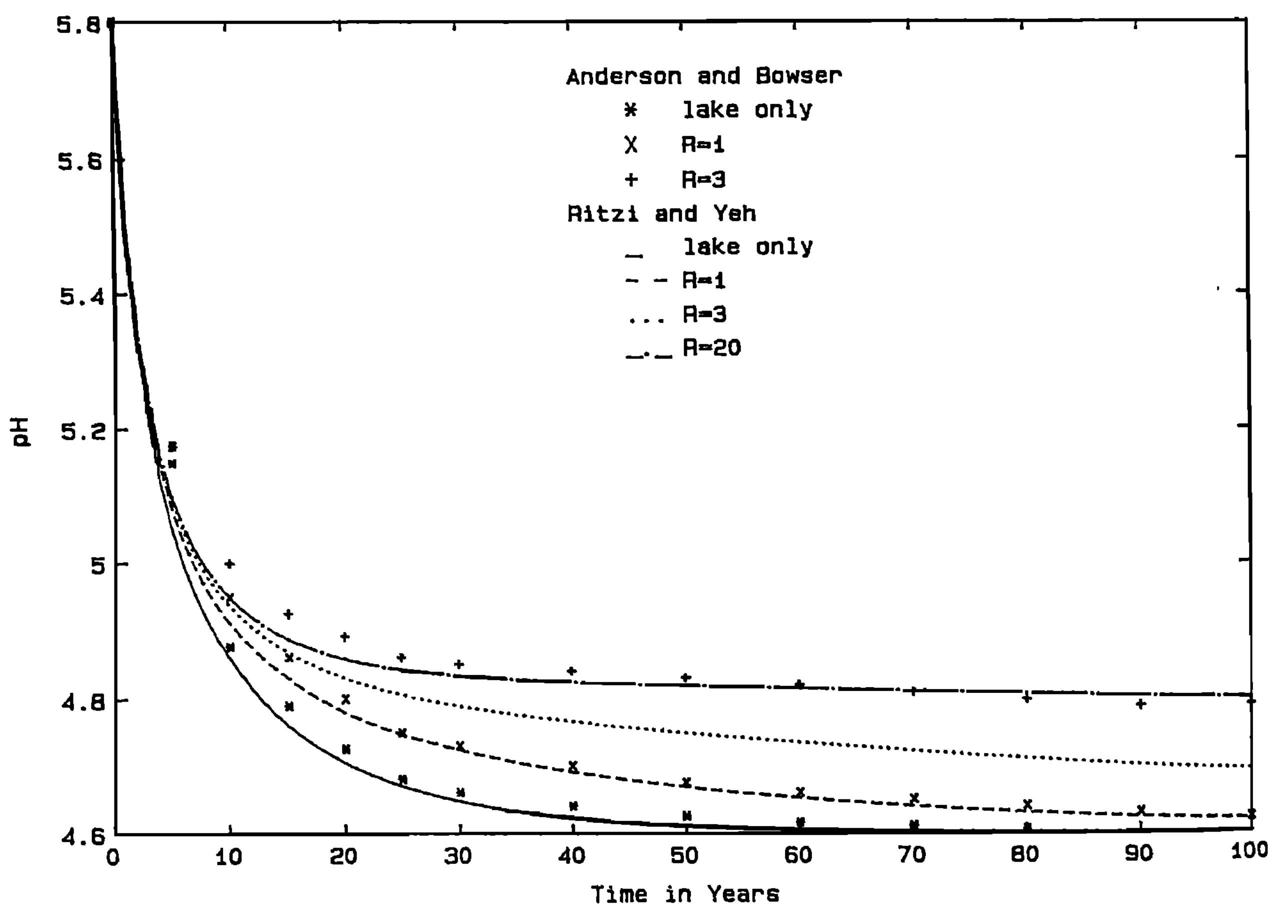

Fig. 1. Lake pH with time without groundwater input (solid curve), with groundwater input with $R = 1$ (dashed curve), $R = 3$ (dotted curve), and $R = 20$ (dashed-dotted curve).

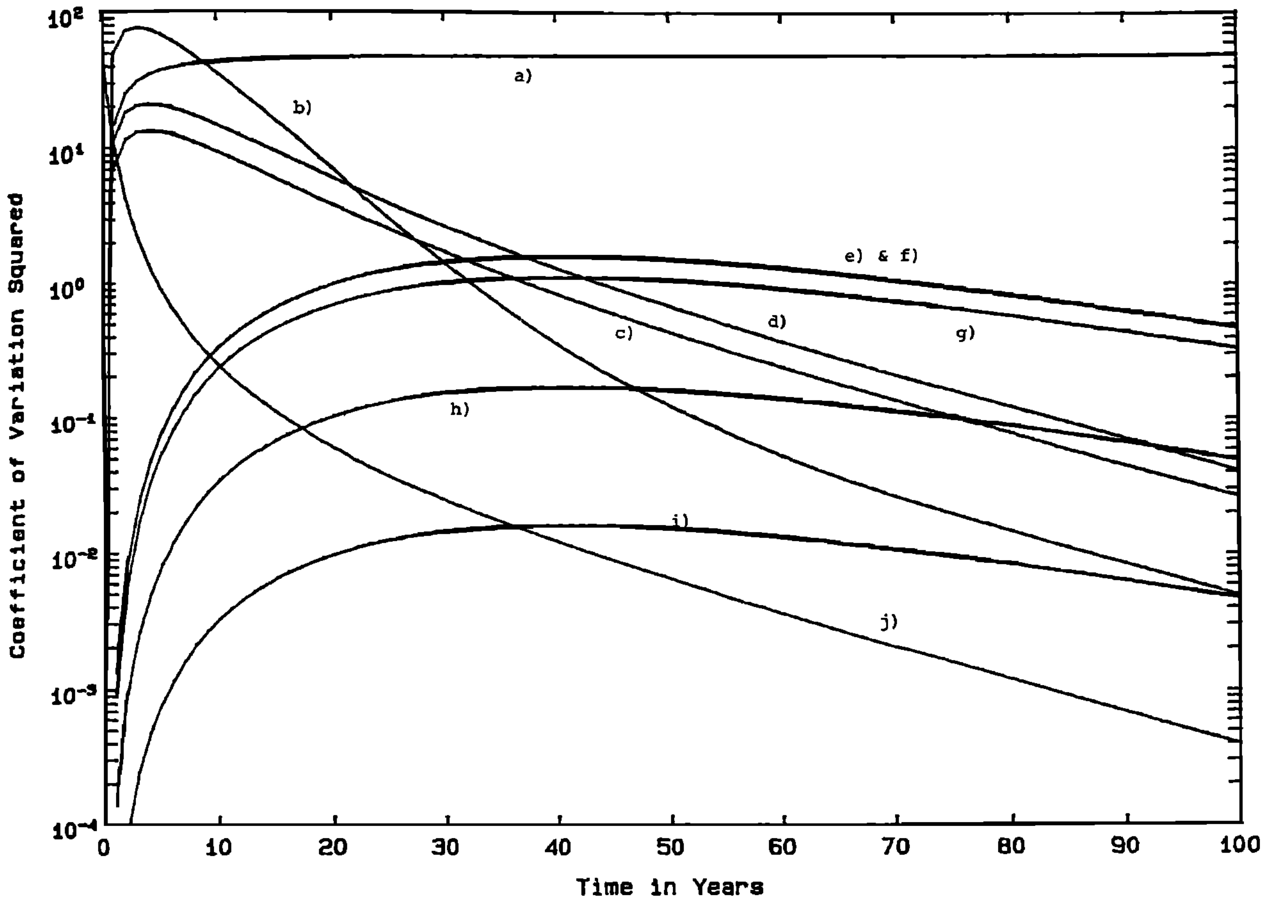

Fig. 2. Coefficient of variation squared plotted against time for each model parameter: (*a*) C_p and C_g, (*b*) V, (*c*) q, (*d*) p, (*e*) h, (*f*) R, (*g*) ε, (*h*) L, (*i*) n, and (*j*) C_l and $C_a(0)$.

perturbation in each individual parameter value θ_i by

$$\text{Var }[g(\theta)]_{\theta_i} = \text{Var }(\theta_i)\left[\left.\frac{\partial g(\theta)}{\partial \theta_i}\right|_{m_{\theta_i}}\right]^2 \tag{8}$$

where m_{θ_i} is the mean of the parameter θ_i. Using (8) the coefficient of variation, CV, is computed, considering all of the successive parameters. A plot of CV^2 for each parameter considered against time shows the relative sensitivity of the model output to each parameter in time.

Figure 1 illustrates the model output for four cases: (1) the lake model with no groundwater input, (2) the case of groundwater input with the retardation factor equal to unity (i.e., no retardation), (3) the case of $R = 3$, and finally, (4) the case of $R = 20$. The general behaviors predicted by the lumped parameter model are in close agreement with those of AB. However, discrepancy becomes significant at large times for $R = 3$. To obtain a similar result as the case of $R = 3$ in the work by AB, a value of $R = 20$ has to be used. The results of our sensitivity analysis (see following discussion) indicate minor model sensitivity to aquifer parameters including R, and support our results. We postulate that one of the possible reasons for this discrepancy in lake concentrations when considering retardation may be errors due to discretization in the numerical simulations by AB.

Figure 2 shows the coefficient of variation squared versus time for each of the model parameters from a first-order sensitivity analysis, again using the numerical values of Table 1. We see that within the first 20 years of lake evolution, the system is most sensitive to lake volume, initial concentration from which the sensitivity decays with time, and recharge concentration from which the sensitivity grows with time. Of secondary significance are the rainfall and groundwater recharge volumes to the lake. The aquifer parameters R, h, L, ε, and n are of minor significance. It should be stated that these results only apply to this particular point in parameter space. The sensitivities may be different, should different values for the parameters be considered.

Through the lumped parameter analysis, the behavior of a lake-aquifer system in response to increased acid deposition is observed to be the same as in the analysis of AB. Their conclusion that "even when chemical reactions are ignored, transit through the groundwater system causes a delay in arrival of acid to the lake and slows the acidification process" thus can be restated here. However, in the previous analysis considerable computational effort must have been expended in obtaining numerical solutions, while the present analysis is accomplishable in a relatively short period of time with a simple hand calculator. Additionally, a quantitative analysis of the sensitivity of lake response to the aquifer system can be easily obtained through a first-order analysis. Also, the results from numerical models may be subject to numerical errors due to discretization.

Finally, we wish to stress that while the lumped parameter model with well-mixed assumption may not be appropriate for every problem of our interests, the analysis can be done quickly on a hand calculator as an initial approach to complex problems. Through such simple initial analysis, important insight into the design and use of subsequently more complex models may result in some overall conservation of time in the modeling effort.

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R. W. Ritzi, Jr. and T.-C. J. Yeh, Department of Hydrology and Water Resources. The University of Arizona, Tucson, AZ 85721.

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