Analyses of Slug Tests and Hydraulic Conductivity Variations in the Near Field of a Two-Well Tracer Experiment Site

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Abstract

Three-hundred eight slug tests were conducted in a 5 × 5 m area in a coastal, sandy aquifer at the Georgetown site in South Carolina to characterize three-dimensional aquifer heterogeneity. Methods developed by Hvorslev, Bouwer and Rice, and Cooper et al. were employed to estimate hydraulic conductivity values from the slug test data. These three methods produced similar spatial distributions of the hydraulic conductivity but quite different values. Overall, the method of Cooper et al. produces higher conductivity values in high permeability zones but lower values in low permeability areas than the Hvorslev method. Variances of the natural log of conductivity values derived from Hvorslev's and Bouwer and Rice's methods agree with those in the other aquifers under similar depositional environments. However, the variance calculated for the data based on the method of Cooper et al. appears unreasonably large. Despite these differences, histograms of the three sets of conductivity values exhibit bimodal distributions, reflecting stratification of the aquifer. Geostatistical analyses show that correlation lengths and statistical anisotropy of the hydraulic conductivity spatial structure varies with depth.

Introduction

Aquifers are inherently heterogeneous. A knowledge of the spatial distribution of aquifer hydrologic properties is essential in predicting the migration of contaminants in the subsurface. Many field studies have been conducted in the last decade to investigate the effect of hydrologic heterogeneities on solute transport in large-scale aquifers (e.g., Pickens and Grisak, 1981; Molz et al., 1986 and 1989; Freyberg, 1986; Sudicky, 1986; Killey and Moltyaner, 1988; LeBlanc et al., 1991; Boggs et al., 1992; and Jensen et al., 1993). Most of these studies focused on the hydraulic conductivity variability over the scale of tens and hundreds of meters. Little attention has been given to the heterogeneity at the near field (a few meters from the tracer source) which appears to explain our difficulties in interpreting tracer breakthrough data obtained from a small scale two-well test at the Georgetown site (Mas-Pla et al., 1992), and the split of tracer plumes observed in a tracer experiment (Yeh et al., 1995).

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The slug test has been one of the most commonly used techniques for in situ measurement of the hydraulic conductivity in both confined aquifers (e.g., Molz et al., 1989; and Melville et al., 1991), and unconfined aquifers (Hinsby et al., 1992; and Rehfeldt et al., 1992). The usefulness of slug tests as compared with other techniques (such as flowmeter tests, tracer tests, and laboratory permeameter tests) was discussed by Herzog and Morse (1984), Molz et al. (1989), Rehfeldt et al. (1992), and Welby (1992). Molz et al. (1989) reported that conductivity values derived from slug tests were different from those obtained by the other techniques. Nevertheless, the spatial patterns of hydraulic conductivity variation in the field estimated by these different methods were similar. In general, slug tests are easier and less expensive than the others.

For analyzing slug test data, several mathematical models have been developed in the past. The method by Cooper et al. (1967) is developed strictly for two-dimensional radial flow in confined aquifers and it can be used to estimate both hydraulic conductivity and storativity. Methods by Hvorslev (1951) and by Bouwer and Rice (1976) are suitable for three-dimensional flow fields, but they can be used to estimate the hydraulic conductivity only. The Bouwer and Rice method was specifically developed for slug tests in unconfined aquifers.

Because of the differences between the three models, numerous studies have been devoted to investigating the reliability of these models. Numerical simulations performed by Melville et al. (1991) showed that estimates of conductivity by Cooper's method were larger than those estimated by the other two methods. More recently, Butler et al. (1994) conducted detailed numerical simulations of slug tests in perfectly stratified aquifers. They reported that the Hvorslev model can provide acceptable parameter estimates (within 20% of the actual conductivity) for aspect ratios (the ratio of the test interval to the radius of the screen in the test interval) between 3 and 300. In addition, Cooper's method provides better estimates than the Hvorslev model at large aspect ratios.

Using a semianalytical solution, Hyder et al. (1994) investigated the error of hydraulic conductivity estimates associated with the Hvorslev and Cooper methods. They concluded that the Hvorslev method is more suitable for small storage parameters, whereas the Cooper method provides better estimates for large values. With respect to the Bouwer and Rice model, Hyder and Butler (1995) stated that for slug tests in unconfined, homogeneous, isotropic aquifers the model provides estimates within 30% of the actual field values.

The overall objective of this study is to present our extensive three-dimensional hydraulic conductivity characterization at a near-field of the Georgetown site, South Carolina. This extensive site characterization is a part of our efforts to investigate the mechanisms controlling the mobilization of natural organic matter (NOM) in the aquifer (Mas-Pla, 1993). In this paper, we specifically discuss our design of the slug test for the three-dimensional site characterization. In contrast to previous studies of slug tests, we use our extensive field data sets to provide a quantitative comparison of the methods for interpreting slug test results developed by Hvorslev, Cooper et al., and Bouwer and Rice under realistic field conditions. Finally, we discuss our statistical analysis of the hydraulic conductivity data sets to characterize the spatial variability at the experimental site. The use of the conductivity data set in the prediction of tracer plumes at the Georgetown site is reported by Yeh et al. (1995).

Field Site and Experimental Methods

The Georgetown site is located in a coastal, sandy aquifer approximately 3 m thick, with distinct stratification, and bounded by a low permeability clay layer at its bottom. The uppermost layer is approximately 1 m thick and consists of a fine, loamy sand with some clay (9% by weight) and roots. Below this layer there exists a zone of gleyed sand with a 4% clay content, which comprises most of the saturated thickness of the aquifer. The deepest part of the aquifer is a thin layer of medium-coarse sand, consisting of clear quartz with a clay content less than 2%, and ranging from 0.15 to 0.30 m thick. The water table is approximately 1 m below ground level.

Thirty-two wells were installed over a 5 × 5 meters area at the field site for measuring hydraulic conductivity (Figure 1). Each well consisted of a 2.54 cm diameter PVC pipe, fully screened from 1 m below the surface to the bottom of the aquifer. These wells were installed by manually driving the pipe into the soil. Because of the sandy nature of the aquifer and the small diameter of the pipes, the disturbed annulus created by the installation is believed to be small. After installation, these wells were developed by pumping and surging. The slug test was selected as the method for measuring hydraulic conductivity because of the diameter of the wells, the simplicity, and the low cost of the instrumentation for the slug test. The apparatus for the slug test consisted of a water reservoir, three air venting valves, and a PVC pipe that connects the reservoir through a flexible tube (Figure 2). At the end of the PVC pipe, twelve 0.63 cm holes were drilled over an approximately 15 cm section and a screen was installed. Three O-rings were installed at the top and bottom of the screened interval as packers to isolate the flow. The PVC pipe can be placed at any given location along the well.

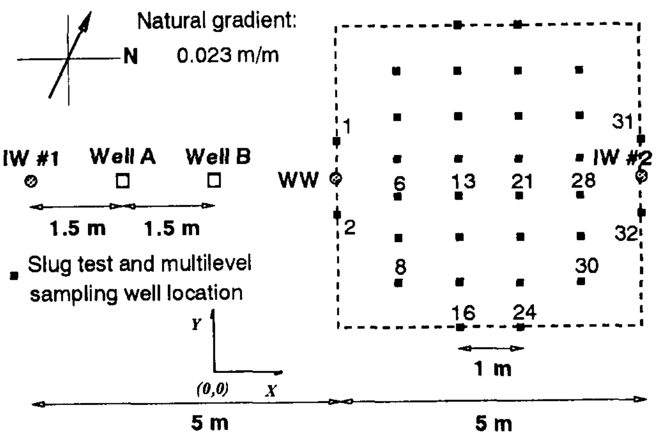

Fig. 1. Well locations of the slug tests at the Georgetown site.

The slug test procedure consisted of (1) placing the screened tip at a desired depth, (2) pumping water (approximately 5 liters) from the aquifer through the PVC pipe up to Valve #2 level (see Figure 2), (3) closing Valves #2 and 3 and opening Valve #1 to allow water from the reservoir to enter the system and vent the entrapped air, (4) allowing the aquifer to equilibrate for 30 minutes, then (5) opening Valve #3 to allow water to flow back to the aquifer. The decline of the water level in the reservoir was then recorded by a video camera as a function of time. The camera was mounted on a tripod at an elevation equal to the mid point of the reservoir. The camera recorded the water level, indicated by a rule mounted beside the reservoir, and the time displayed on a digital stop watch. The recording started just prior to opening Valve #3 so that the record of decrease of water level can be recovered from the tape using a VCR that allowed single frame viewing. The recording rate of 30 frames per second

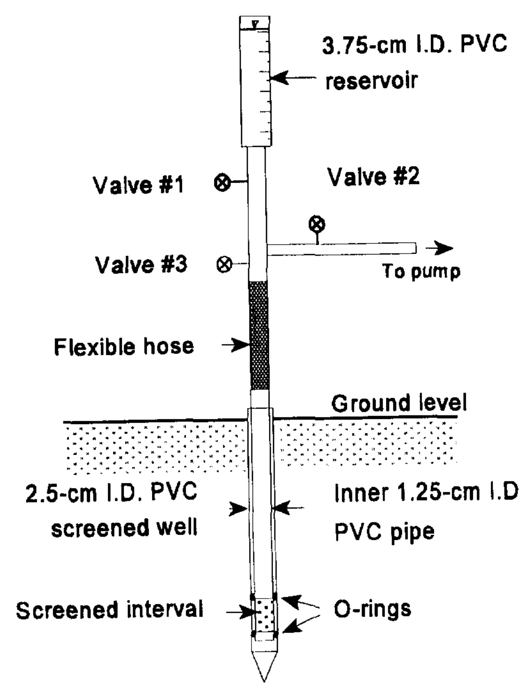

Fig. 2. The design of the slug test apparatus (drawn not to scale).

allowed times to be determined at an accuracy of <0.05 second. For some instances, the use of the video camera also allowed us to perform two slug tests simultaneously at a significantly lower cost than the use of a data acquisition system. At each well, the slug test was conducted at 11 depths at 15 cm intervals.

Methods of Analysis of Slug Test Data

Methods developed by Hvorslev (1951), Bouwer and Rice (1976), and Cooper et al. (1967), were employed for analyzing our slug test data. All three methods assume aquifer homogeneity and isotropy. A brief description of each method is given in the following paragraphs.

The Hvorslev (1951) Method. This method states that the rate of flow from the well, q, at any time t is proportional to the hydraulic conductivity, K, of the aquifer and to the decline of the head level in the well. By neglecting the storage effect of the aquifer, the hydraulic conductivity of the aquifer around the slug test portion can be estimated by

$$K = 2.303 \frac{A}{F} \frac{\log[H_1/H_2]}{t_2 - t_1} \qquad (1)$$

where H_1 and H_2 are the heads recorded at times t_1 and t_2, respectively. F is a shape factor, which depends on the geometry and dimensions of the well intake and its location within the aquifer.

The shape factor F chosen for this analysis corresponds to the case of a point piezometer as illustrated in Figure 12-8 or 18-G by Hvorslev (1951), and it can be expressed as

$$F = 2\pi L / \left[\ln\left(L/2r_w + [1 + (L/2r_w)^2]^{1/2}\right)\right] \qquad (2)$$

where L is the length of the screen, and r_w is its radius. This formula assumes that the length of the screened interval of the slug test is less than one-tenth of the saturated thickness of the aquifer, and the flow regime resulting from the slug test is symmetrical about a horizontal plane through the center of the well.

The Bouwer and Rice (1976) Method. This model resembles the Hvorslev model [equation (1)]. The difference is the shape factor, where Bouwer and Rice include the radius of influence or effective radius, R_e, over which the hydraulic head difference is dissipated. This shape factor is given by

$$F = 2\pi L/(\ln R_e/r_w) \qquad (3)$$

Using a resistance network analog model, Bouwer and Rice (1976) developed an empirical relation between the effective radius, R_e, thickness of the aquifer, b, depth of the well, d, length of the screened interval, L, and the radius of the screen, r_w. This empirical relation is expressed as

$$\ln(R_e/r_w) = \left[\frac{1.1}{\ln d/r_w} + \frac{A + B\ln(b - d/r_w)}{L/r_w}\right]^{-1} \qquad (4)$$

where A and B are dimensionless coefficients, which are functions of L/r_w, and are given in Bouwer and Rice (1976). Bouwer and Rice (1976) also reported that the values of $\ln R_e/r_w$ calculated by equation (4) are within 10% of the actual value as evaluated by the analog model if $L > 0.4d$, and within 25% if $L \ll d$ (e.g., $L = 0.1d$). Butler et al. (1993) reported that the Bouwer and Rice method provides estimates that are within 30% of the hydraulic conductivity of the formation for aspect ratios commonly employed in the field.

The Cooper et al. (1967) Method. This method is based on an analytical solution of the governing equation for two-dimensional radial flow toward/from a fully penetrating well in a confined aquifer. Cooper et al. (1967) derived the solution to these equations and provided a set of type curves that allow the estimation of the transmissivity and the storativity of the aquifer. They reported that the estimate of transmissivity is not very sensitive to matching procedure, and the reliability of estimate of storativity is somewhat questionable. The hydraulic conductivity can be calculated by the relationship $K = T/b$, where b is the thickness of the aquifer, and in this study b was taken as the length of the screened interval. A study by Dax (1987) reported that as the ratio of the aquifer thickness to well radius increases, Cooper's solution can also be used in unconfined aquifers.

To apply the Hvorslev, and Bouwer and Rice methods, the log of water level was plotted against time; most of these plots exhibited a linear relationship between water level and time. As a result, a linear regression analysis was carried out to determine the slope. However, the very early time portion of some data sets deviated from such a linear relationship, probably showing the effect of the disturbed annulus created by the installation of the well (Bouwer, 1989) and/or the effects of storage at early time. It was not considered in the regression. Equation (1) was then used to estimate the hydraulic conductivity with the shape factor given for the test apparatus. During the application of the Bouwer and Rice method, effective radii were calculated using equation (4).

The Cooper et al. method was applied to the data through the use of a nonlinear optimization routine (Heidari and Hemmat, 1992), which automatically determines the best fit T and S values. During the optimization, the S value was constrained within the range between 0 and 1.

Geostatistical Analysis of Hydraulic Conductivity Data

A geostatistical analysis was conducted, using hydraulic conductivity data obtained from the slug tests to quantify the spatial variability of the hydraulic conductivity at the field site. This involves the estimation of the distribution, mean and variance, and the spatial correlation structure of the conductivity. The sample or experimental semivariogram (hereafter simply referred to as the variogram) was employed to characterize the correlation structure. Because of the clear stratification observed from well logs and the complication of a three-dimensional variogram analysis, our study of the correlation structure focused on two-dimensional horizontal (X-Y) and vertical (X-Z and Y-Z) planes. The existence of deterministic trends of the conductivity data was investigated using the computer package BLUEPACK-3D, developed at the École des Mines de Paris, before the semivariogram analysis.

At first, horizontal variograms were estimated for each layer corresponding to the depth where the slug test was conducted at each well. Adjacent layers with similar variances are then considered as the same geological unit (see Table 2). The variograms of adjacent layers with similar variances were then averaged to obtain a mean variogram to avoid the small numbers of pairs in the analysis of a single layer. As a result, the 11 layers were lumped into five geological units, namely, 1, 2, and 3; 4 and 5; 6 and 7; 8 and 9; and 10 and 11. The mean variogram for each group was estimated as (Journel and Huijbregts, 1978)

$$\bar{\gamma}(h) = \sum_{k=1}^{m} N_k(h)\, \gamma_k^*(h) \Big/ \sum_{k=1}^{m} N_k(h) \quad (5)$$

where $\gamma^*(h)$ is the variogram value, $N(h)$ is the number of pairs at lag h, where $N_k(h)$ is the number of pairs at lag h for variogram k, and m is the number of layers in each unit.

Omnidirectional and directional horizontal mean variograms were then calculated. Directional mean variograms were used to explore the possible statistical anisotropy of the correlation structure on lnK. These mean variograms were calculated along the 0, 45, 90, and 135° directions, measured clockwise from the y-axis in Figure 1, and with a tolerance angle of 45°. The reason for using such a broad tolerance angle was to guarantee that we had sufficient pairs of conductivity data during the variogram estimation. The minimum lag distance was set to 0.6 m, the shortest distance between wells.

Directional variograms for the vertical planes were analyzed along the vertical and horizontal directions only, as suggested by the layered structure of the aquifer. Similarly, a tolerance angle of 45° was maintained to enclose a sufficiently large number of data points during the analysis of variograms in vertical planes. Note that the number of data points in these vertical planes was notably larger than in the horizontal ones, and the minimum lag in the analysis was set to 0.15-0.20 m. One-dimensional vertical variograms were estimated separately for each well, using a lag of 0.15 m and then were averaged using equation (5).

To obtain variogram sill and range, theoretical variogram models were fitted to the experimental variogram by minimizing the sum of the squares of the difference between the experimental value and the model value for each lag distance h. Differences between the experimental and model values were weighted according to the number of data pairs (A. Guzmán, personal communication). A theoretical exponential variogram model was selected to represent the horizontal correlation structure of lnK;

$$\gamma(h) = c[1 - \exp(-3\, h/a)] \quad (6)$$

where c is the sill of the variogram, which in theory should correspond to the sample variance, and a is the range, which describes the extent of spatial correlation of lnK. The nugget term is not included as it did not significantly modify the estimation of variogram parameters. For the vertical direction, a Gaussian variogram model with a nugget effect was chosen based on the shape of the experimental variogram,

$$\gamma(h) = c_0 + c[1 - \exp(-3h^2/a^2)], \quad h > 0 \quad (7)$$

where c_0 is the nugget effect.

Results and Discussion

Analysis of Slug Tests

Although a total of 308 slug tests were conducted, results of 248 tests were selected for the analysis, and the others were discarded because of unreasonable water decline curves (large fluctuations in the water level), possibly caused by operational errors during the tests. In addition, 19 out of the 248 tests were also discarded for the Cooper et al. analysis due to an unsatisfactory fit between the mathematical model and the data (i.e., the general trend of the data is quite different from that of the model).

The frequency distributions of the conductivity values estimated by the three different methods are illustrated in Figure 3. The conductivity values derived from the three different methods show bimodal distributions, in contrast to the lognormal distribution commonly reported in the literature (e.g., Sudicky, 1986). The bimodal distribution consists of two lognormal distributions, reflecting the geological layering of the aquifer (gleyed sand of the upper portion of the aquifer and the coarse sand at the bottom). A summary of the statistics of the conductivity distributions is presented in Table 1. It shows that the geometric mean of K is similar for all methods, ranging from 1.08×10^{-5} and 1.65×10^{-5} m/s. The minimum and maximum values of lnK show that the Hvorslev and the Bouwer and Rice methods gave similar extreme values, whereas the Cooper et al. method offered

Table 1. Summary of the Hydraulic Conductivity Statistics

	Hvorslev	*Bouwer & Rice*	*Cooper et al.*
Number of data	248	248	229
Geometric mean, m/s	1.65×10^{-5}	1.34×10^{-5}	1.08×10^{-5}
Mean of lnK, m/s	-11.01	-11.22	-11.43
Variance of lnK	1.40	1.62	5.58
Minimum lnK, m/s	-13.84	-14.12	-16.30
Maximum lnK, m/s	-8.89	-8.85	-7.12

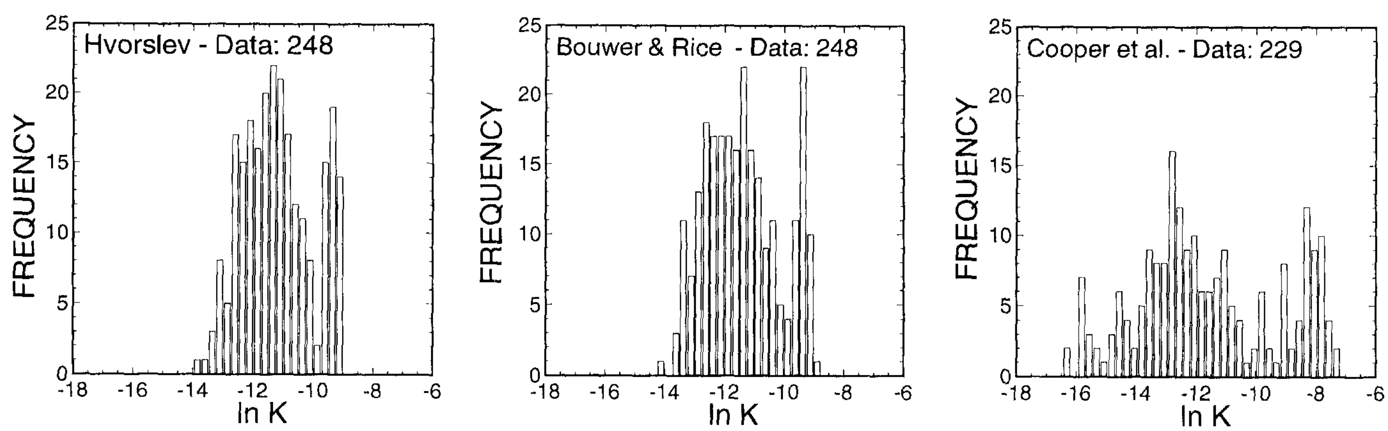

Fig. 3. Frequency distributions of the hydraulic conductivity (lnK) estimates using the Hvorslev, Bouwer and Rice, and Cooper et al. methods. K is in m/s.

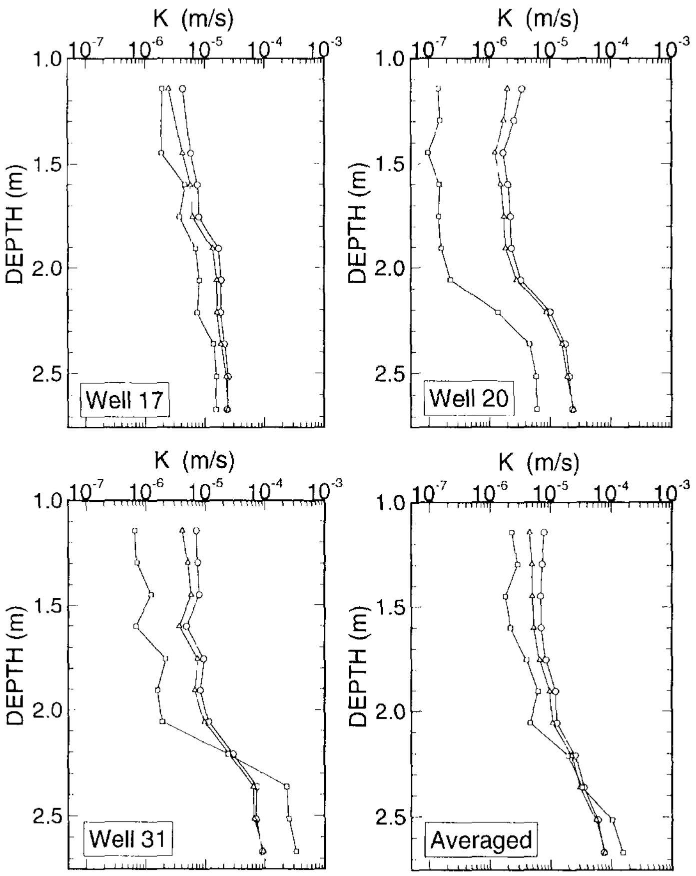

Fig. 4. Hydraulic conductivity ($\log_{10}K$) profiles at three wells (a, b, and c), and the averaged conductivity profile (d). Circles: Hvorslev methods; Triangles: Bouwer and Rice method; and Squares: Cooper et al. method. Well locations are given in Figure 1.

a broader range of results. Consequently, the lnK variance of the Cooper et al. method is significantly larger than those derived from the other two methods. In fact, the variances of lnK for the data sets based on the Hvorslev and the Bouwer and Rice methods agree well with those reported by others (e.g., see Table 2 in Gelhar, 1986) for similar materials in similar depositional environments. The high variance, 5.58, of the data set based on the Cooper et al. method appears unreasonable for such a sandy aquifer.

Figures 4a, b, and c show the profiles of conductivity values derived from the three different methods at different locations. These conductivity profiles display a similar trend: an increase in conductivity values with depth, and a sharp increase in conductivity at a depth of approximately 2.25 meters below the surface. Averaged conductivity values for each layer show an identical profile (Figure 4d). This trend agrees well with the grain size profile based on core samples from the well bores. The Hvorslev and the Bouwer and Rice methods give very similar values, but the method of Cooper et al. produces conductivity values greater or smaller by an order of magnitude than the other two methods.

The mean and variance of hydraulic conductivity values for each layer are given in Table 2. Again, the mean conductivities estimated by the Hvorslev and Bouwer and Rice methods are similar, but the conductivity values estimated by the former method are smaller than those by the latter by a factor of approximately two for all depths. However, the relationship between these values and those based on the Cooper method varies. For instance, lnK profiles are similar in some locations (Figure 4a), whereas they differ by one order of magnitude in others (Figure 4b). Furthermore, lnK value derived from the Cooper method is smaller in the uppermost layers at most locations, and is larger in the lower layers in comparison with that from the Hvorslev method (Figure 4c).

The different value of the variance of lnK in each layer reflects different degrees of lateral variation of the conductivity between layers. Figure 5 shows the plan views of the distribution of lnK derived from the Hvorslev method at six different depths. As illustrated in the figure, the spatial distribution of conductivity at some depths is almost uniform (e.g., at 2.65 m depth, $\sigma^2 = 0.28$), while in other layers lnK values range from −9.5 to −12 (e.g., at 2.05 m depth, $\sigma^2 = 0.81$). These lateral variations of conductivity, mainly in the upper layers, can be attributed to root disturbances and other unknown soil structures. Figure 6 presents a similar plot using the lnK data obtained from the Cooper et al. method. The lnK distributions in a cross section between the injection and withdrawal wells (Figure 7) again show that, though depicting a similar structure, the method of Cooper et al. produces a larger contrast in hydraulic conductivity values than the Hvorslev method, resulting in a higher variance of lnK (Table 2).

The storativity of the aquifer materials estimated by the Cooper et al. method shows a broad range of values from 10^{-8} to 1. Only about 10% of the results were larger than 0.5. In general, storativity values estimated for the upper half of the aquifer were of the order of 10^{-1}, corresponding to an acceptable value for the aquifer's specific yield. For the lower half, especially for the lowest 0.5 m, the storativity values were notably smaller, with averages of 10^{-2}, values, which are more appropriate for the value of the elastic storativity of the aquifer than of its specific yield. No other methods are available for estimating the storativity of the aquifer from the slug test data, and the validity of these values is difficult to assess.

The high variability of the transmissivity values estimated by the method of Cooper et al. could be attributed to the large range of the S value allowed in the optimization. According to the Cooper et al. type curves for slug tests, low values of S will correspond to large values of T, and vice versa. Fixing the value of S over a narrow range may reduce the spread of T values, and

Table 2. Mean and Variance of LnK at Each Depth

			Hvorslev		*B&R***	*Cooper et al.*	
Layer	*Depth*	*# Data**	*Mean*	*Var.*	*Mean*	*Mean*	*Var.*
1	1.15	17 (11)	−11.75	0.41	−12.30	−12.97	2.12
2	1.30	17 (13)	−11.83	0.44	−12.22	−12.77	1.16
3	1.45	21 (18)	−11.88	0.44	−12.20	−13.21	1.60
4	1.60	25 (22)	−11.87	0.61	−12.15	−13.03	3.49
5	1.75	24 (22)	−11.69	0.73	−11.93	−12.43	3.66
6	1.90	25 (23)	−11.32	1.01	−11.54	−11.98	3.61
7	2.05	23 (24)	−11.28	0.81	−11.44	−12.29	3.64
8	2.20	24	−10.56	0.96	−10.70	−10.84	5.09
9	2.35	24	−10.24	0.92	−10.38	−10.31	5.12
10	2.50	25	−9.72	0.59	−9.79	−9.18	3.46
11	2.65	23	−9.48	0.28	−9.44	−8.77	1.82

Depth is in m, and K is given in m/s.
*-Numbers in parentheses indicate the number of data points for the Cooper et al. method.
**-The lnK variances at each layer for the Hvorslev, and Bouwer and Rice solutions are identical as the ln K estimates only differ by the shape factor F.

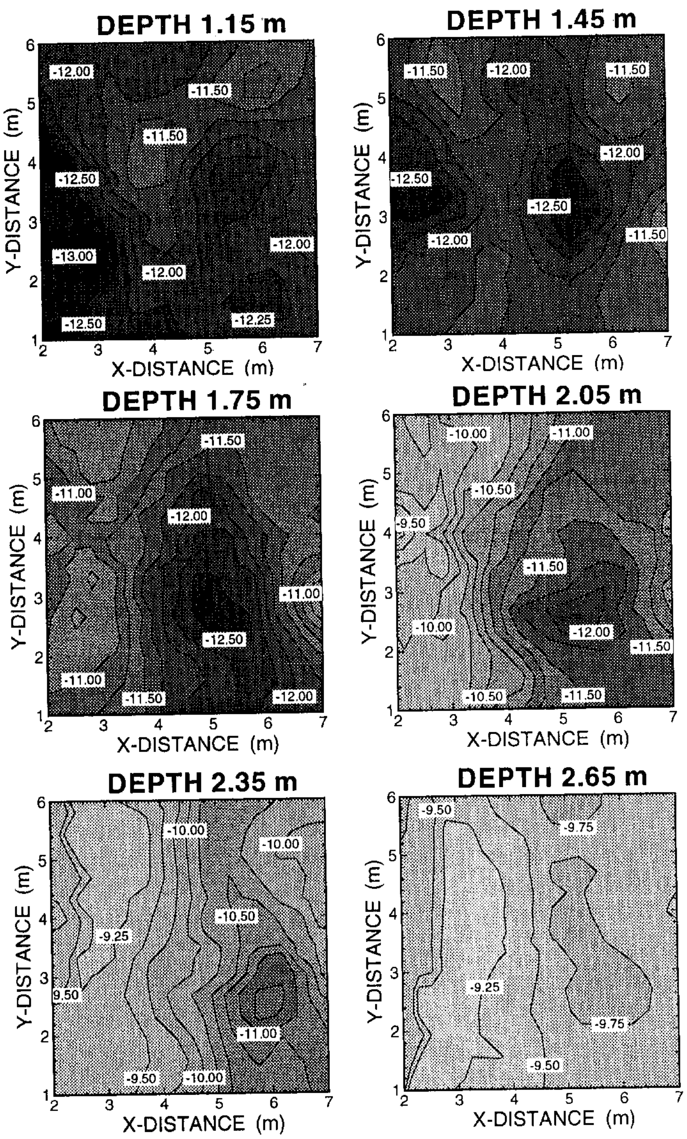

Fig. 5. Plan views of the hydraulic conductivity estimates (lnK) derived from the Hvorslev method, at different depths of the aquifer. The mapped surface corresponds to the dashed square in Figure 1.

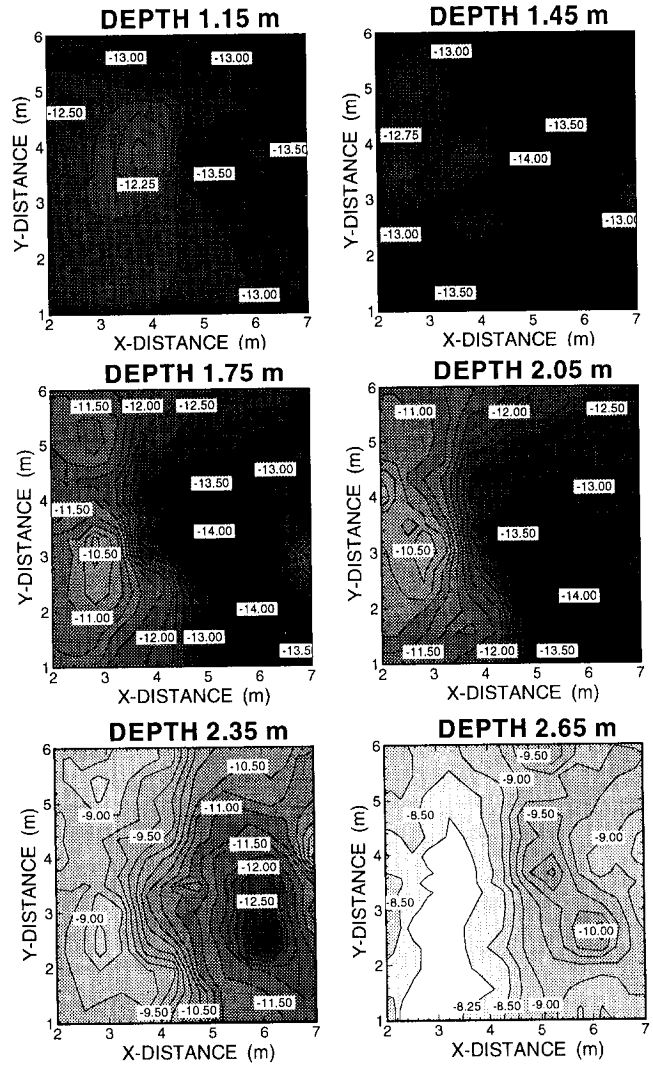

Fig. 6. Plan views of the hydraulic conductivity estimates (lnK) derived from the method by Cooper et al., at different depths of the aquifer. The mapped surface corresponds to the dashed square in Figure 1.

therefore reduce the variances. Since no prior estimates of S were available, fixing the value of S over a narrow range may introduce a systematic bias in the result. Nevertheless, the estimated S values appear to be consistent with those of a sandy aquifer.

A three-dimensional view of the conductivity distribution using the data set derived from the Hvorslev method (Figure 8) summarizes the main feature of the geohydrology of the field site. It shows that the site consists of stratified geological

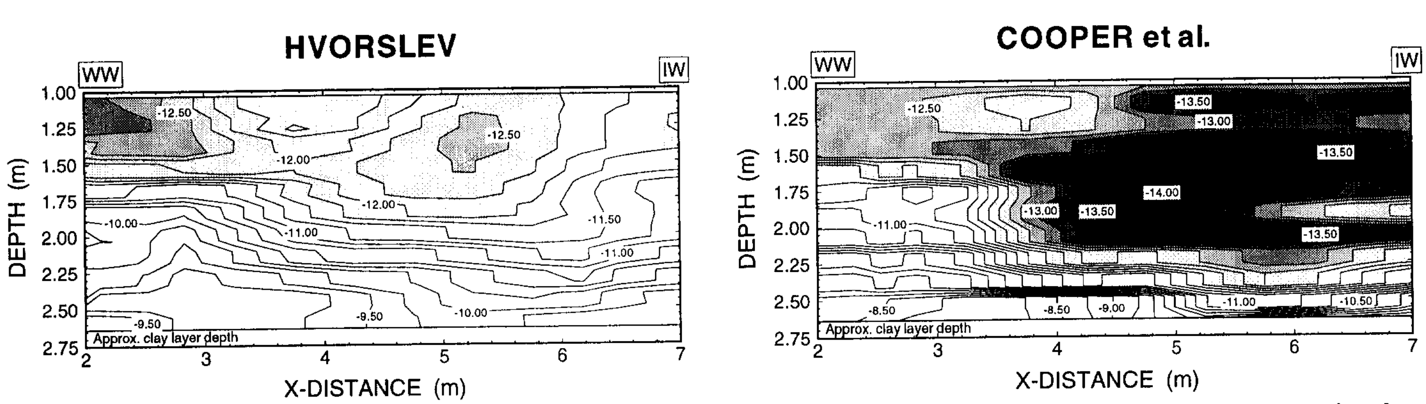

Fig. 7. Distributions of the hydraulic conductivity estimates (lnK) based on the Hvorslev and the Cooper et al. methods of the cross section along the transect between the injection well (IW) and withdrawal well (WW).

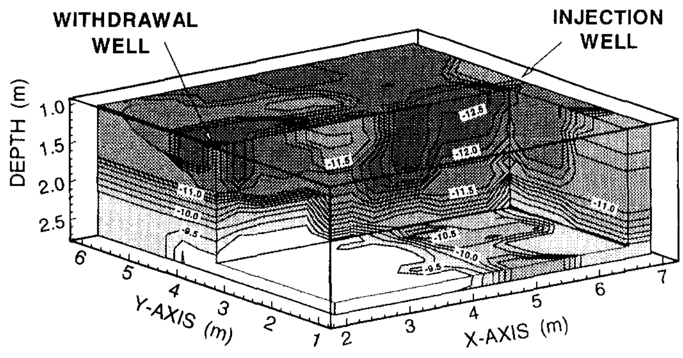

Fig. 8. Three-dimensional view of the estimated hydraulic conductivity (lnK) distribution based on the Hvorslev method. The mapped volume corresponds to the dashed square surface in Figure 1.

materials with low permeability materials near the surface, and high permeability materials at the bottom of the aquifer. The stratification is further complicated by several localized low and high permeability zones between the injection and the withdrawal wells.

Since the conductivity estimates by the Hvorslev, and Bouwer and Rice methods are very similar, the following discussion will focus on differences between estimates by the Hvorslev method and those by Cooper et al. A quantitative comparison between hydraulic conductivity estimates derived from the Hvorslev and the Cooper method is presented in Figure 9, where a linear equation,

$$\ln K_{\mathrm{Cooper}} = 9.276 + 1.901 \ln K_{\mathrm{Hvorslev}} \tag{8}$$

was fitted (r = 0.919). Based on this figure, it is evident that the Cooper et al. method produced larger conductivity values at high permeability zones and lower conductivity values at low permeability zones than the Hvorslev method.

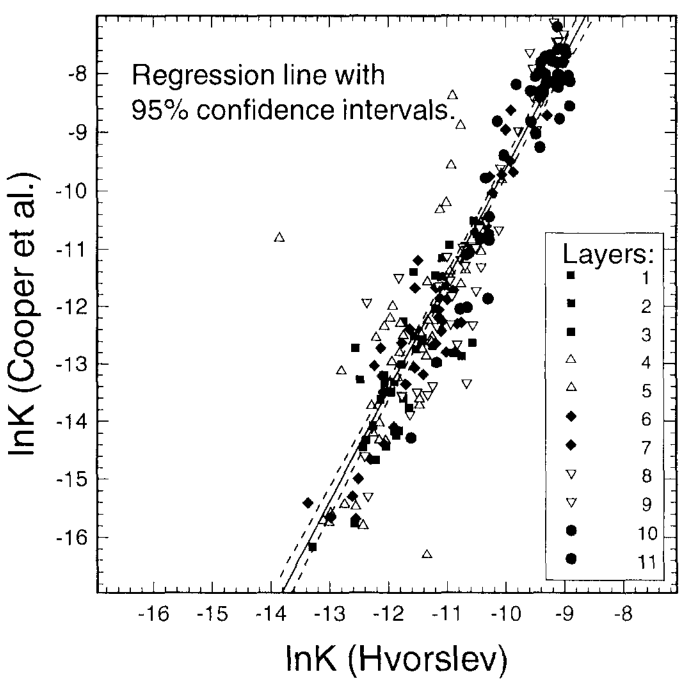

Fig. 9. A comparison of the hydraulic conductivity estimates (lnK) based on the Hvorslev method and the method by Cooper et al. Regression line (solid) and the 95% confidence intervals (dashed).

Due to the differences in hydraulic conductivity estimates by the two methods, one may ask: which data set is the most representative of the hydraulic conductivity of the field site. To address this question, one must first consider the question of whether these methods represent the actual flow system induced by the slug test. Since the slug test was conducted using packers at different depths, it is likely that the outflow from the packered interval creates a three-dimensional ellipsoidal flow field. The Hvorslev method based on the point piezometer geometry is most appropriate for analyzing this type of slug test. On the contrary, the method by Cooper et al. is not suitable since it considers a fully penetrating well in a confined aquifer, and is restricted to a horizontal flow regime.

Furthermore, the method derived by Cooper et al. explicitly includes the storage effect, a result of aquifer matrix and water compressibilities. It provides an additional parameter, storativity, for adjustment during the optimization procedure. Since the response of the aquifer is controlled by the ratio of the transmissivity to storativity, it is likely that different values of transmissivity and storativity, but with the same ratio, may produce similar water level responses during a slug test.

An additional consideration is the consistency between the estimated hydraulic conductivity values and the lithology of the field site. Based on core samples obtained at the field site, these sandy materials do not appear to be highly heterogeneous, at least not to the extent to show about two orders of magnitude variation of K between the top and the bottom of the aquifer, as reflected in the conductivity values derived from the method of Cooper et al. In addition, variances estimated for each layer using the Cooper et al. method are extremely high for a sandy aquifer as compared to the values reported in the literature for similar geological depositional environments. Consequently, we believe that the conductivity values resulting from the Hvorslev method are a more reasonable representation of the aquifer properties at the Georgetown site. It should be pointed out that interference between wells during the simultaneous slug tests might have occurred. Although we have not addressed this problem, our successful numerical simulation of tracer plumes (Yeh et al., 1995) using the data set derived from the slug tests suggest that the error due to the interference appears to be small.

Several recent studies (Melville et al., 1991; and Butler et al., 1993) of methods for analysis of slug test data reported the Cooper's method should produce conductivity values greater than the method by Hvorslev due to the two-dimensional horizontal flow assumption embedded in the Cooper et al. method. As shown in Figure 9, the conductivity value estimated based on the Cooper approach may be greater or smaller than the one estimated by Hvorslev's approach. Although we do not know exactly the cause of such discrepancies, we believe that our extensive data set provide a realistic comparison of these two methods under field conditions.

Analysis of the Correlation Structure

Horizontal variograms. A trend of order one was identified in each horizontal layer, and it was removed to obtain residuals. Experimental mean variograms using the residuals of lnK were then estimated for each of the five groups of layers. Sill and range values of the omnidirectional and directional variograms are presented in Table 3. The sill value can be considered constant in each direction, but the variation of range values indicates that each group exhibits an anisotropic correlation structure. Also

Table 3. Variogram Analysis of lnK (Horizontal Planes, Residuals)

		0/90		0/45		45/45		90/45		135/45	
Layers	*Depth*	*Sill*	*Range*	*Sill*	*Range*	*Sill*	*Range*	*Sill*	*Range*	*Sill*	*Range*
1-2-3	1.35	0.41	0.36	0.41	0.19	0.45	0.80	0.42	0.17	0.41	0.18
4-5	1.72	0.64	1.78	0.52	1.32	0.57	1.45	0.77	2.25	0.65	2.05
6-7	2.02	1.03	2.62	0.66	1.70	0.88	1.96	1.47	3.64	1.15	4.19
8-9	2.32	0.63	0.83	0.59	1.51	0.63	0.89	0.69	0.23	0.62	1.06
10-11	2.62	0.34	1.05	0.39	3.67	0.33	1.08	0.37	0.15	0.33	1.40
All	—	0.61	1.50	0.50	1.32	0.58	1.38	0.72	1.78	0.60	1.75

Legend: 0/45 reads as "variogram along direction 0° and tolerance angle 45°." In particular, the variogram 0/90 corresponds to the omnidirectional variogram.
Directions are measured clockwise from the y-axis (Figure 1).
Depth represents the average depth of each layer group. Depth and range are given in m.

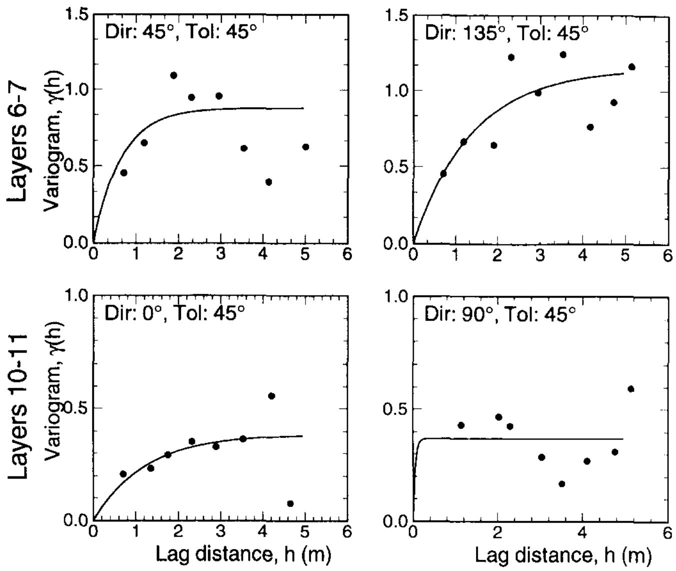

Fig. 10. Experimental (dots) and modeled (solid line) variograms of lnK, for layers 6-7 (depths 1.90 and 2.05 m) and 10-11 (depths 2.50 and 2.65 m).

shown in the table is that the anisotropy of the directional variograms and its orientation varies with depth, indicating possible changes of the sedimentary structure of the site. Such changes in anisotropy at depth are clear at the depths 2.02 m (layers 6-7) and 2.62 m (layers 10-11) (see Figure 10). The major axis of the anisotropy (with the largest range) appears oriented along the 135° direction at the depth 2.02 m, and along the 0° direction at depth 2.62 m. These results seem consistent with the lnK distributions (Figure 5).

As shown in Table 3, the maximum statistical anisotropy ratio was estimated to be 2.14 for group 6-7, and 24.5 for group 10-11. The large anisotropy ratio for the bottom layer group may be attributed to the small range value (0.15 m) which is likely to be a poor and unreliable estimate due to the lack of data at short distances (less than the sample interval). Although no evidence in the conductivity maps (Figure 5) supports such a high anisotropy ratio, the map shows the 90° direction as the minor axis of anisotropy. Small range values (less than the sample interval) at the upper layers can also be attributed to the fact that the theoretical variogram with a zero nugget was used, or the spatial correlation structure in the upper layers simply cannot be detected by such a sparse sampling interval.

The hole effect has been identified in the variograms (e.g., layers 10-11 in Figure 10), which reflect a pseudo-periodicity on the horizontal plane (Journel and Huijbregts, 1978). This interpretation is consistent with the "valleys" and "ranges" observed in the distribution of the conductivity. Finally, Table 3 also shows the sill and ranges of the mean directional variograms for the 11-layer average.

Vertical variograms (X-Z and Y-Z planes). Although a weak first-order trend was identified for the vertical plane variograms, there was no clear evidence that residuals would offer a better representation of the spatial correlation than the original data. Therefore, the original data were used, and the resultant sills and ranges are presented in Table 4.

Table 4. Variogram Analysis of lnK (Vertical X-Z and Y-Z Planes)

	lnK		0/90			0/45			90/45			
Plane	*Mean*	*Var.*	*Model*	*Sill*	*Range*	*Model*	*Sill*	*Range*	*Model*	*Nug.*	*Sill*	*Range*
x = 4 m	-10.81	1.35	Exp	1.56	1.62	Exp	1.64	1.90	Gauss	0.20	2.47	1.36
x = 5 m	-11.61	0.98	Exp	1.09	1.23	Exp	1.06	1.90	Gauss	0.03	1.70	1.05
x = 6 m	-11.31	0.92	Exp	1.22	1.90	Exp	1.32	4.21	Gauss	0.31	3.97	3.52
Averaged	-6.62	1.22	Exp	1.34	1.86	Exp	1.40	3.85	Gauss	0.07	2.14	1.36
y = 1.75 m	-6.58	1.22	*	*	*	*	*	*	Gauss	0.53	4.87	1.82
y = 2.50 m	-11.40	1.73	*	*	*	*	*	*	Gauss	0.44	2.57	0.74
y = 3.20 m	-11.15	2.05	*	*	*	*	*	*	Gauss	0.26	4.72	1.41
y = 3.80 m	-11.25	1.55	*	*	*	*	*	*	Gauss	0.09	3.21	1.90
y = 4.50 m	-10.99	0.95	*	*	*	*	*	*	Gauss	0.22	1.75	1.17
y = 5.25 m	-10.78	1.03	*	*	*	*	*	*	Gauss	0.62	2.54	1.67
Averaged	-6.51	1.51	*	*	*	*	*	*	Gauss	0.06	2.60	1.20

Legend: 0/45 reads as "variogram along direction 0° and tolerance angle 45°." In particular, the variogram 0/90 corresponds to the omnidirectional variogram. K was initially given in m/s. Range is given in m. * indicates that no variograms could be modeled along that direction (see text). Coordinates along x and y are consistent with Figure 1.

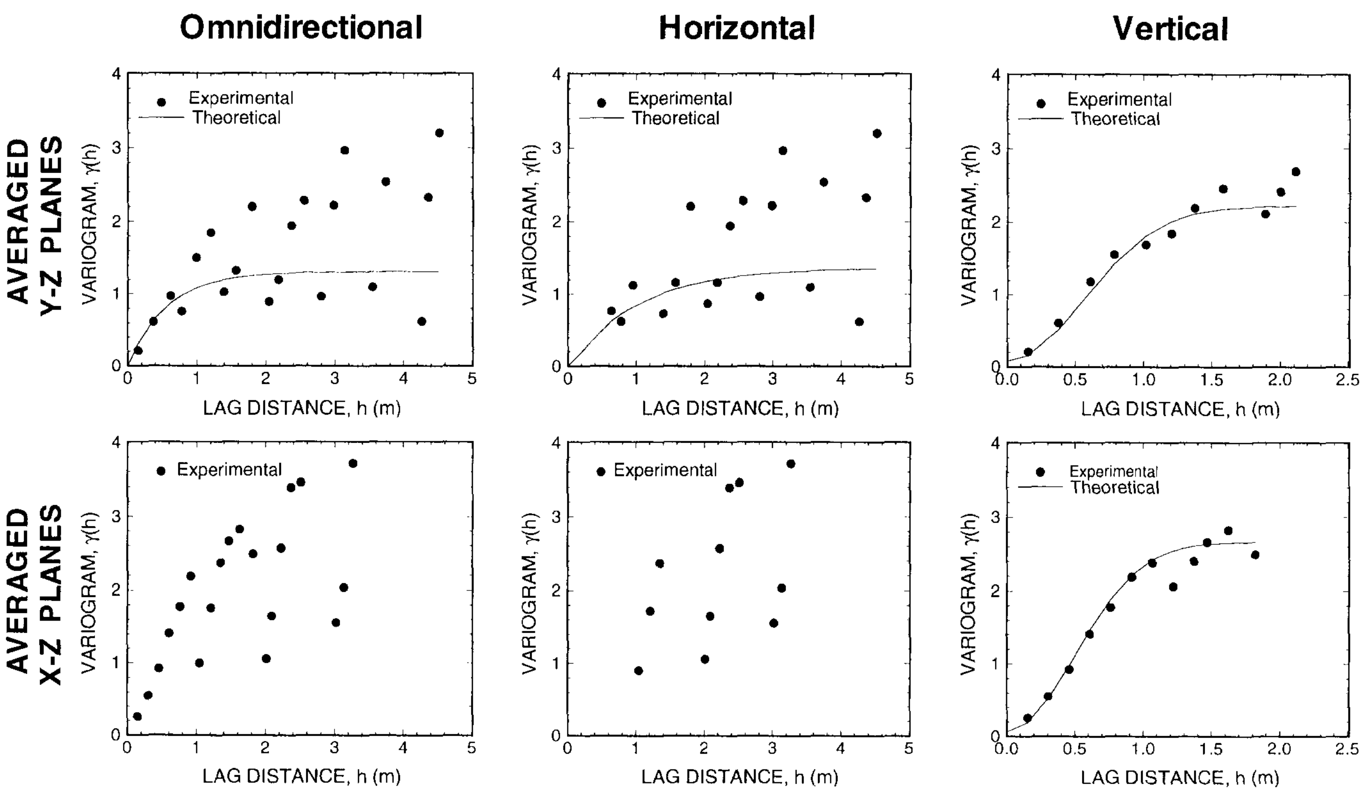

Fig. 11. Experimental (dots) and modeled (solid line) averaged variograms for planes Y-Z (constant X) and planes X-Z (constant Y).

Averaged variograms for planes of constant X and constant Y were plotted in Figure 11. From this figure, experimental directional variograms along the horizontal direction (X and Y direction) were quite scattered, and an exponential model was used to fit them in the case of the Y-Z planes. No further attempts were made to fit the horizontal variograms corresponding to the X-Z planes, where no correlation structure is seen. Conversely, directional variograms along the vertical direction (90/45) were satisfactorily described by a Gaussian model. Journel and Huijbregts (1978) suggested that a Gaussian variogram model represents a continuous spatial variability of conductivity with distance. The smoothly varying conductivity profiles in Figure 4d support their suggestion. Omnidirectional variograms shown in Figure 11 integrate the characteristics observed in the directional variograms.

Finally, a mean one-directional vertical variogram, estimated from the original lnK values, presents a clear Gaussian model shape too with a sill value of 2.55, a nugget of 0.15, and a range of 2.10 m. This variogram is extremely similar to the ones estimated for the vertical planes (Figure 11), illustrating the smooth vertical variability of conductivity across the aquifer.

Conclusions

Based on the above analysis, we conclude that the conductivity values derived from the Hvorslev, and Bouwer and Rice methods are the most representative of the sandy aquifer at the Georgetown field site. Results of three-dimensional numerical simulation (Yeh et al., 1995) of the migration of tracer plumes in the aquifer, using the estimated conductivity data sets, indeed, support this conclusion.

The aquifer at the Georgetown site can be portrayed as a layered aquifer with the highest conductive layer at its bottom. Such a contrast in conductivity distributions between the upper and the bottom aquifer layers produces the bimodal distribution of lnK for the entire aquifer. Geostatistical analysis of the hydraulic conductivity (lnK) data suggests that ranges of variograms and directions of anisotropy vary among layers, indicating variations of the sedimentary structure with depth.

Finally, such complex distributions of lnK and the variable statistical anisotropy (or spatial structure) of the conductivity among layers may explain our difficulties in applying the classical two-well analysis (equivalent to macrodispersion approach) to our previous tracer test result (Mas-Pla et al., 1992). More importantly, we raise doubts on the assumption of aquifer homogeneity in the interpretation of small-scale field tracer tests. A more appropriate interpretation may have to rely on a three-dimensional numerical simulation which includes a detailed characterization of the field site heterogeneity, as applied by Yeh et al. (1995).

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