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Stochastic Analysis of Unsaturated Flow in Heterogeneous Soils
3. Observations and Applications

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Results of stochastic theory for flow in heterogeneous soils are analyzed by comparisons with laboratory experiments and field observations, and through applications examples. The two key theoretical results are (1) the variability of capillary pressure or moisture content increases when mean capillary pressure increases and (2) the anisotropy ratio (horizontal/vertical) of effective (mean) unsaturated hydraulic conductivity increases when mean capillary pressure increases or mean moisture content decreases. Comparisons with the field data on moisture content and capillary pressure variability show trends similar to those predicted by the theory. Calculations of hydraulic conductivity anisotropy based on two actual soils show that the variations in soil texture produce large changes in anisotropy as the mean capillary pressure changes. Several previously reported field observations and laboratory experiments support the theoretical finding of a capillary pressure dependent hydraulic anisotropy for unsaturated flow. The importance of this anisotropy effect in applications involving groundwater recharge, irrigation, surface runoff generation, and waste isolation is discussed.

INTRODUCTION

The theory developed in parts 1 and 2 of this series [*Yeh et al.*, this issue (*a*, *b*)] indicated that the capillary pressure head variance increases with its mean value, i.e., the variance becomes large as the soil becomes dryer. Furthermore, the effective unsaturated hydraulic conductivity depends on the mean gradient, the orientation of stratification, and the correlation scales, and most importantly, its anisotropy varies substantially as mean capillary pressure changes if the variance of the parameter α is large.

Before discussing the importance of these results we will first apply the results of the analysis to a field situation where relatively large amounts of soil hydrologic data are observed so that the stochastic results can be tested. Then the effects of variability of soil hydraulic properties on the anisotropy of the unsaturated hydraulic conductivity are demonstrated using some available field data. Finally, some implications of moisture-dependent anisotropy in vadose zone hydrology and its applications in waste isolation problems and design of capillary barriers are discussed.

COMPARISON OF MOISTURE CONTENT VARIANCE

To illustrate the predictive capability of the theoretical result of the head variance, the theoretical result, equation (26*b*) in part 2, is applied to a field situation where some of the necessary parameters have been observed. *Nielsen et al.* [1973] have observed a large number of hydraulic conductivity-soil moisture relationships and soil-water retention data over a 150-ha field near Fresno, California. The soil is generally classified as the Panoche soil series, which has uniform profiles but a wide range of textures including loam, clay loam, silty clay, and silty clay loam. The statistical homogeneity assumption employed in the theoretical analysis restricts the use of the entire data set and only data collected in the silty clay loam (nine plots, six depths, and 30.48-cm intervals) are used in the calculation of soil moisture variation.

In order to apply the theory to the field situation the parameters, σ_f^2, σ_a^2, λ_1, and λ_2, have to be estimated from the available data set. Since the hydraulic conductivity data are measured in terms of soil moisture content Θ, a translation of $K - \Theta$ to $K - \psi$ is necessary for this calculation. This translation is carried out by a direct conversion of Θ to ψ according to the soil-water retention curve observed at each depth of each plot. By fitting the exponential hydraulic conductivity model, (1) of part 1 [*Yeh et al.*, this issue (*a*)], to the converted $K - \psi$ relation data, the α (slope) and ln K_s (intercept) of each ln $K - \psi$ curve was determined. Estimates of σ_f^2, σ_a^2, and the mean a for this soil are 2.47, 0.000067 cm^{-2}, and 0.0294 cm^{-1}, respectively. The data are not adequate to estimate the correlation scales of this particular field and the horizontal and vertical correlation scales are assumed to be 40 and 1.0 m, respectively, based on information reported by *Bakr* [1976] and *Viera et al.* [1981].

Head variance is predicted by using the theory equation (26*b*) in part 2 [*Yeh et al.*, this issue (*b*)] with $J = 1$ and the estimated parameters. To obtain a direct comparison of observed data to the calculated result, the calculated head variance is related to the variance of soil-moisture content by a simple linear relationship:

$$\theta = c\psi + d \qquad (1)$$

where θ is moisture content, and ψ is capillary pressure. Both θ and ψ are random quantities in (1). In addition, c and d also can vary from soil to soil and thus are taken to be random variables. The variance of θ is approximated by linearizing the $c\psi$ term in (1) and ignoring the correlation between ψ and c or d. Thus

$$\text{Var}(\theta) \cong [E(c)]^2 \text{ Var}(\psi) + [E(\psi)]^2 \text{ Var}(c) + \text{Var}(d) + 2E(\psi) \text{ Cov}(c, d) \qquad (2)$$

Estimates of $E(c)$, Var (c) etc. were obtained by using each of the 54 moisture retention curves, fixing ψ, and obtaining the associated θ value. This procedure yields the controlled independent variable model described in *Graybill* [1961]

$$\theta_0 = c\psi_0 + d + \varepsilon$$

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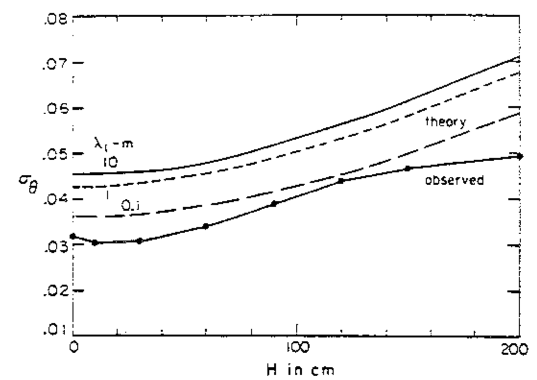

Fig. 1. Comparison of observed (.) and calculated standard deviation of moisture content as a function of mean capillary pressure head H; $\lambda/\lambda_1 = 40$.

where ε is a random quantity with $E(\varepsilon) = 0$, Var $(\varepsilon) = \sigma_\varepsilon^2$. By using the 54 curves in the pressure range 0–200 cm, a standard regression procedure yields values of c and d, and then the means, variances, and covariances for these two data sets were then calculated. The dominant term in (2) was $[E(c)]^2$ Var (ψ) with all other terms contributing very little to Var (θ). The estimated variance and mean of c of the 54 linear models are 4.83×10^{-8} cm^{-2} and -6.02×10^{-4} cm^{-1}, respectively. The variance of d is found to be 1.06×10^{-3}; the covariance of c and d is -6.53×10^{-7} cm^{-1}. The average value of the error variance in the regression analyses is 5.63×10^{-4}.

The results are illustrated in Figure 1, where the standard deviation of calculated moisture contents at different mean capillary pressure heads are plotted along with the standard deviations of observed moisture data. Results obtained for other values of the vertical scale λ_1 and λ_2 (same aspect ratio) are also illustrated in Figure 1. The theoretical calculations are subjective because λ_1, λ_2, and the correlation between the a and f processes of this field are not known. The field situation may also involve significant unsteady flow effects which are not accounted for in the theory. In addition, the exponential model does not accurately describe the $K - \psi$ relation for each observed data set over the entire range of pressure head. The assumption of a linear model for capillary pressure head and moisture content could further affect the actual behavior of the relation. However, it is interesting that the theory predicts the observed magnitudes of σ_θ with reasonable values of λ_1 and λ_2. Moreover, the theory predicts the general trend of increasing moisture content variance with mean capillary pressure.

To further illustrate the dependence of capillary pressure variation on mean pressure, capillary pressure measurements collected by *Fritton* [1981] in a 2-ha corn field are analyzed. Estimated variances of the capillary pressure head are listed in Table 1 along with its mean values. Similar trends are noted for the data collected by *Yeh* [1982] in an alfalfa field (see Table 1). Both observed variations of capillary pressure qualitatively confirm the theoretical result that the head variance grows with the mean capillary pressure. In other words, the drier the soil is, the higher the capillary pressure head variance.

ANISOTROPY OF UNSATURATED HYDRAULIC CONDUCTIVITY

The theory developed in part 2 [*Yeh et al.*, this issue (*b*)] predicts that the anisotropy of effective conductivity is a function of mean capillary pressure. What is the magnitude of anisotropy in a realistic soil formation at low saturation? To answer this question the anisotropy ratios of two soils, Panoche silty clay loam [*Nielsen et al.*, 1973] and Maddock sandy loam [*Carvello et al.*, 1976], are evaluated using the theoretical model for the special case with a unit vertical mean hydraulic gradient. Representative behavior of the hydraulic conductivity of the two soils as a function of capillary pressure head is shown in Figures 2 and 3. Note that the variation of the slope α of the ln $K - \psi$ curves of the Maddock sandy loam is more drastic as compared to the Panoche soil.

The procedure used in estimating the required parameters of Maddock sandy loam for this analysis is essentially the same as the previous example for the Panoche soil. However, the number of data points available in this case is much less (two plots and seven depths). Figures 2 and 3 include the estimated values of the parameters of the two soils needed to evaluate the anisotropy. The variance of ln K_s and α and the mean of α for the sandy loam are significantly greater than those of the silty clay loam. The value of σ_f^2 for the sandy loam is unusually large. This large variation can be attributed to extrapolation of the unsaturated conductivity to the saturated condition based on the simple exponential conductivity model. However, the large variation of f does not significantly affect the results for high mean capillary presssures, since the contribution from the mean pressure head is much greater than that of the variance of saturated hydraulic conductivity.

The anisotropy ratios for these two soils with $\rho = \lambda_2/\lambda_1 = 40$ are derived using the same method as (34) in part 2 [*Yeh et al.*, this issue (*b*)] where the variance and the cross-covariance terms for finite ρ are evaluated numerically [*Yeh*, 1982]. Figure 4 illustrates the mean capillary pressure dependence of the anisotropy ratio of these two soils. The figure shows that as mean capillary pressure increases, the anisotropy of Maddock sandy loam is far greater than the anisotropy of the Panoche silty clay loam. This difference can be understood easily if one examines the anisotropy formula equation (34*b*) of part 2 derived from the theory for the special case where $J_1 = 1$, $J_2 = J_3 = 0$, $\Theta = 0$, and $\rho \to \infty$. That result shows that the anisotropy ratio grows exponentially with the exponent $(\sigma_f^2 + \sigma_a^2 H^2)/(1 + A\lambda)$. Because the sandy loam has a large σ_a^2 (about 2 orders of magnitude larger than that of silty clay loam), the conductivity anisotropy of sandy loam grows rapidly as mean capillary pressure increases. On the other hand, the rate of growth of the anisotropy of Panoche silty clay loam is insignificant at this pressure range.

In explanation of this anisotropy phenomenon, consider the sandy loam as a medium consisting of layers of soils of two distinct textures: clay and sand. If sand overlies a clay layer, after a steady state infiltration is established, a perched water table can develop in the sand just above the boundary with

TABLE 1. Mean and Standard Deviation of Soil Capillary Pressure in Some Fields

Fritton [1981]		*Yeh* [1982]	
Mean, cm	Standard Deviation, cm	Mean, cm	Standard Deviation, cm
1200	470	162	22
2000	1000	185	33
4400	2800	186	35
8980	7140	187	36
		325	80
		334	137
		336	95
		402	170

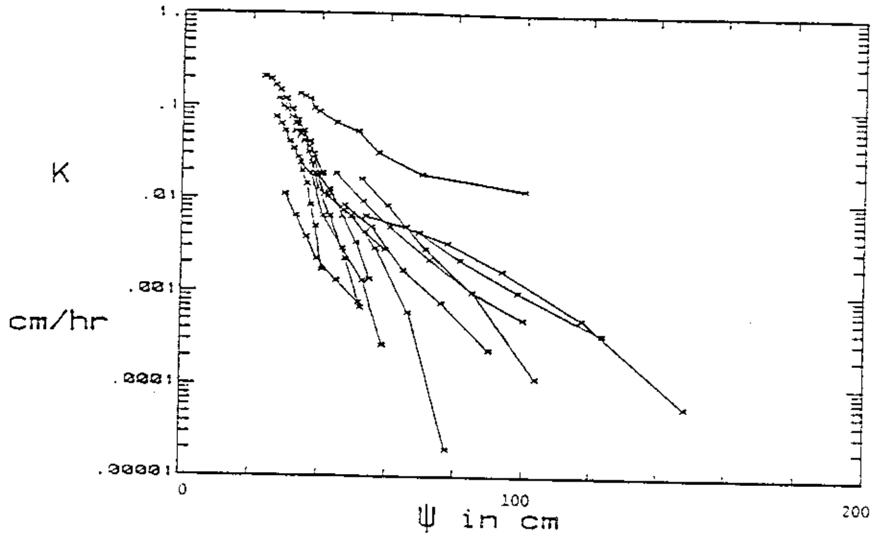

Fig. 2. Observed $K - \psi$ relationships of Maddock sandy loam [data from *Carvello et al.*, 1976]; $\sigma_f^2 = 7.45$, $\sigma_a^2 = 0.006\ cm^{-2}$, and $A = 0.147\ cm^{-1}$.

the less permeable clay layer. Thus the lateral flow can be greater than the vertical flow. If a clay layer overlies a sand layer then, since there is a rapid reduction of hydraulic conductivity of sand as capillary pressure increases, at some pressure ranges the unsaturated conductivity of clay is greater than that of sand. Vertical movement of water from the clay layer can be restricted because of the small conductivity of sand at this pressure range. Thus water is confined to the clay layer and tends to disperse laterally in the clay due to capillarity. As capillary pressure increases, the conductivity of sand is further reduced, the conductivity of the clay may remain relatively constant, and consequently the anisotropy becomes more significant. In contrast, the silty clay loam has a uniformly textured material throughout the profile as is evident from Figure 3. The contrast in unsaturated hydraulic conductivity of each layer is relatively small for the whole range of capillary pressure. Water can more easily propagate downward and lateral movement of moisture would be limited.

One possible way to estimate the effective conductivity anisotropy is to take the ratio of arithmetic mean and harmonic mean of conductivity values. We refer to this method as the "direct average method." For saturated flow the arithmetic mean represents the effective conductivity in the direction parallel to the bedding of a layered composite medium, whereas the harmonic means represents that in the direction normal to the bedding [*Bear*, 1972; *Gutjahr et al.*, 1978; *Gelhar and Axness*, 1983]. By applying this simple concept, the hydraulic conductivities of each soil, reconstructed from the previously estimated parameters and the exponential conductivity model, are arithmetically averaged at each pressure to obtain a mean $K - \psi$ curve that represents the unsaturated conductivity in the direction parallel to the bedding. Similarly, the harmonic mean of the hydraulic conductivities at each pressure head is used to represent the mean hydraulic conductivity in the direction normal to the bedding at that pressure; hence each soil has two $K - \psi$ curves to describe the unsatu-

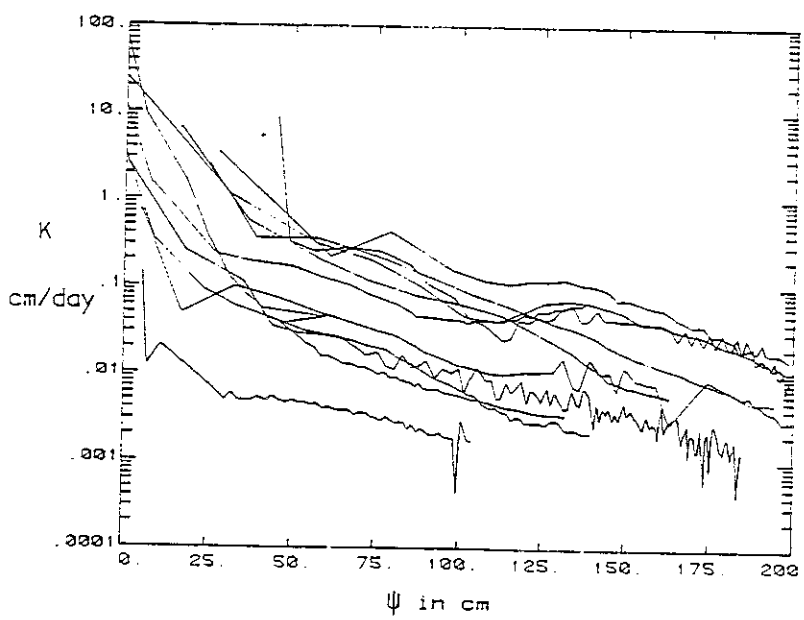

Fig. 3. $K - \psi$ relationships of Panoche silty clay loam [data from *Nielsen et al.*, 1973]; $\sigma_f^2 = 2.47$, $\sigma_a^2 = 0.000067\ cm^{-2}$, and $A = 0.029\ cm^{-1}$.

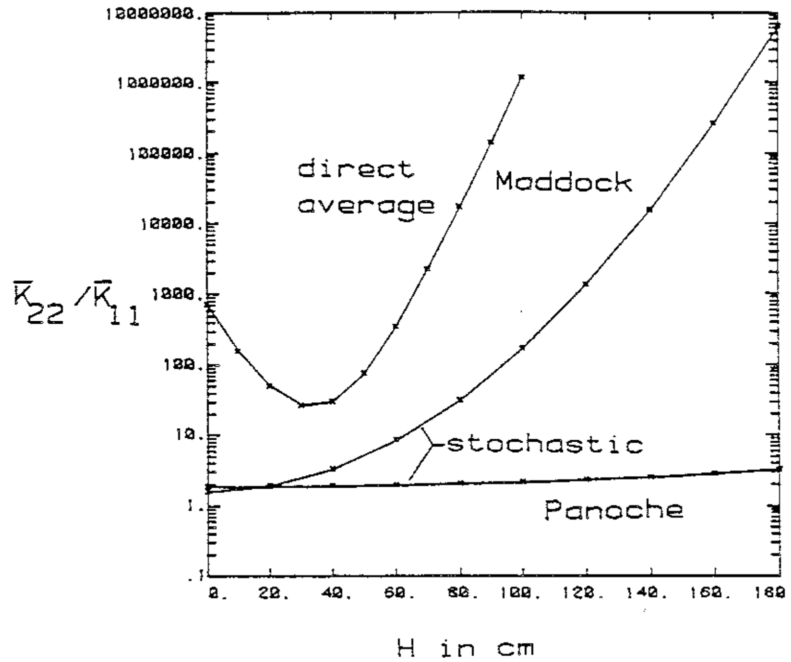

Fig. 4. Comparison of the anisotropy ratios obtained from the stochastic theory and the direct average methods for the Maddock sandy loam and for Panoche silty clay loam; H is the mean capillary pressure head.

rated hydraulic conductivities in two principle directions. The anisotropy ratio, which is the ratio of the two curves, is depicted in Figure 4 for the Maddock sandy loam. The anisotropy ratio for both the direct average method and the stochastic theory show similar dependence of the anisotropy ratio on the mean capillary pressure. However, the direct average method yields a consistently higher anisotropy in the range of interest.

To explain this difference, note that if ln K is normally distributed, the harmonic mean and the arithmetic mean can be expressed

$$K_h = K_m \exp(-\sigma_{\ln K}^2/2) \qquad K_a = K_m \exp(\sigma_{\ln K}^2/2)$$

in which $\sigma_{\ln K}^2$ is the variance of the unsaturated conductivity, and K_m is the geometric mean of the unsaturated hydraulic conductivity. The theory behind the direct average method can be unravelled from the evaluation of $\sigma_{\ln K}^2$. From (1) and (4) in part 1 [*Yeh et al.*, this issue (*a*)], ln K can be expressed as

$$\ln K = F + f - (A + a)\psi \tag{3}$$

if ψ is considered to be nonrandom. Subtracting the mean of (3) from (3), the perturbation equation becomes

$$\ln K - E[\ln K] = f - a\psi$$

correspondingly,

$$\sigma_{\ln K}^2 = E[(f - a\psi)^2]$$
$$= \sigma_f^2 - 2\,\mathrm{Cov}\,(a, f)\psi + \sigma_a^2\psi^2$$

and if a and f are uncorrelated,

$$\sigma_{\ln K}^2 = \sigma_f^2 + \sigma_a^2\psi^2$$

Thus the anisotropy ratio is

$$K_a/K_h = \exp(\sigma_{\ln K}^2) = \exp(\sigma_f^2 + \sigma_a^2\psi^2) \tag{4}$$

By comparing (4) with (34*b*) in part 2 [*Yeh et al.*, this issue (*b*)],

$$\bar{K}_{22}/\bar{K}_{11} = \exp[(\sigma_f^2 + \sigma_a^2H^2)/(1 + A\lambda_1)]$$

which is the result for the special case where $J = 1$, $\theta = 0$, and $\rho \rightarrow \infty$, the reason that the direct average method overestimates the anisotropy ratio becomes clear. The stochastic theory produces a smaller anisotropy due to the presence of the denominator $(1 + A\lambda_1)$. The direct average method assumes that capillary pressure head is a deterministic variable. In reality, ψ is a stochastic process, as has been observed in field studies [*Yeh*, 1982]. Furthermore, ψ is directly related to the flow regime, which is subject to the influence of the correlation scale of the soil property. Therefore without solving the governing flow equation, the direct average method fails to provide a realistic result.

Although field data for a quantitative comparison of anisotropy with the theoretical anisotropy results are not available, some field observations of moisture movement in the vadose zone do indicate qualitative confirmation of the saturation dependence of unsaturated anisotropy. *Crosby et al.* [1968, 1971*a*, *b*] reported findings of comprehensive field observations of pollutant migration from a septic tank drain field in glacial outwash deposits in the Spokane Valley, Washington. Results of soil sample analysis indicate that chemical pollutants travel with moisture fronts but the moisture is limited to the upper 20–25 feet (609.6–762 cm), approximately one quarter of the thickness of the unsaturated zone in the area. To explain the extremely dry condition at the depth below 25 feet (762 cm), they state that "the drain field waters quite obviously must be dispersing laterally in the finest interbed in response to capillary gradients exceeding the gravitational potential". This lateral flow postulation seems consistent with the anisotropy derived from the theory.

Several other studies report observations that support the results presented here. *Routson et al.* [1979] monitored high-level radioactive waste leakage in the unsaturated zone that showed extensive lateral (horizontal) moisture movement but limited vertical movement. *Sanai et al.* [1974] indicated that field observations in the loess-loam area north of Beer-Sheba, Israel showed that rain infiltrates the soil to a limited depth with no net recharge of the groundwater, and they suggested a

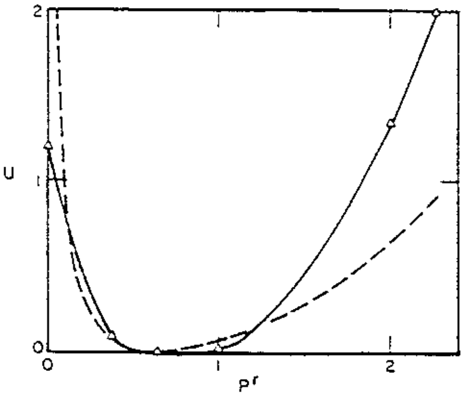

Fig. 5. Comparison of anisotropy coefficient U derived from the stochastic result (dashed curve) and the deterministic analysis of *Zaslavsky and Sinai* [1981*b*], Figure 4 (solid curve with triangles); $P_r = P/K_{s1}$ and $U = (\bar{K}_{22}'/\bar{K}_{11}' - 1)$.

lateral flow effect in the unsaturated soil. *Dirksen* [1978] reported laboratory experiments on unsaturated flow from buried line sources which "... exhibit varying degrees of anisotropy, with the hydraulic conductivity being greater horizontally than vertically"; the anisotropy apparently reflected the unintentional layering when the model was packed with a uniform fine sand.

Qualitative evidence of the anisotropy effect in unsaturated flow is provided by the laboratory experiments conducted by *Palmquist and Johnson* [1962] in which downward movement of water from disposal pits was simulated by using initially dry glass bead models. The first model consisted of a homogeneous bed of glass beads; the second model consisted of three layers of glass beads of silt size separated by two layers of medium size. Water was introduced through a crib at the top of the models and after a steady state was reached, the wetted area in the first model was found to be confined to a relatively narrow vertical column. The second model took a much longer time to reach a steady state; the flow was retarded by the coarse beds and the wetting front moved horizontally until enough hydraulic head had been built up to allow water to move through the coarse beds. As a result of stratification of different grain size of material, the wetted area became much greater in lateral extent.

Direct evidence of the saturation dependent anisotropy was reported by *Corey and Rathjens* [1956]. In their Figure 4, relative permeability of oil and gas measurements on the Berea sandstone core demonstrate that anisotropy is a function of oil saturation. They stated that

> "... Berea sandstone which, when dry or fully saturated, appears to be homogeneous and isotropic. When the material is partially desaturated, however, thin and regular spaced strata are apparent. Moreover, the air permeability of the dry core is about twice as great parallel to the bedding planes as perpendicular to them. Evidently, the material is quite uniform, but it is not isotropic. The effect of the anisotropy is to increase greatly the critical gas saturation and to make the oil relative permeability curve steeper when flow is across the bedding planes".

Although the scale of the core sample is relatively smaller than the scale of the theory considers, the observation supports the theoretical result.

Zaslavsky and Sinai [1981*a, b*] explore the concept of the anisotropy of unsaturated flow in a stratified soil formation. Based on a theoretical study of steady state infiltration in a two-layered soil, they conclude that the soil actually behaves as an anisotropic medium in which the anisotropy increases with the rate of vertical flow. The horizontal component of the hydraulic conductivity can be several times the vertical one. To show the similarities between the stochastic theory and their results on anisotropy the stochastic result is applied to their study. Their model considers only a two-layered deterministic system, but it does provide a direct comparison of the anisotropy derived from the stochastic analysis.

A medium consisting of two homogeneous soil layers is considered by *Zaslavsky and Sinai* [1981*b*]. The thickness of these two layers are D_1 and D_2 with saturated conductivities K_{s1} and K_{s2}. The relationship of unsaturated conductivity and capillary pressure head was also assumed to be exponential and the α parameters are α_1 and α_2 for the upper and lower soil layers, respectively. The anisotropy of the effective unsaturated conductivity of this composite medium was derived from the analysis of steady state infiltration into the two-layered medium with an inclination angle of θ. The lateral capillary pressure gradient parallel to the layers was assumed to be zero. For small slopes ($\cos\theta \cong 1$, $\sin\theta \ll 1$) their anisotropy coefficient, $U = (\bar{K}_{22}'/\bar{K}_{11}' - 1)$, was predicted as a function of rainfall intensity P. The rainfall intensity was expressed in the normalized form $P^r \equiv P/K_{s1}$, which can be evaluated from the condition of (7) in the work by *Zaslavsky and Sinai* [1981*b*], $P\cos\theta = q_n \equiv q_1' = \bar{K}_{11}'\cos\theta$, since a unit vertical mean hydraulic gradient is assumed. Using (33) of part 2 [*Yeh et al.*, this issue (*b*)] with $\sigma_f^2 + \sigma_a^2 H^2$ replaced by $\sigma_f^2(1 - \zeta H)^2$ and $K_m = K_G \exp(-AH)$, $K_G \cong (K_{s1}K_{s2})^{1/2}$, $g \cong A\lambda_1$, and $a_{11} = \cos\theta$,

$$P^r = \bar{K}_{11}'/K_{s1} = (K_{s2}/K_{s1}) \exp - \left[AH + \frac{\sigma_f^2(1 - \zeta H)^2}{2(1 + A\lambda_1)} \right] \quad (5)$$

Similarly, their anisotropy factor is determined from (34*a*) of part 2 [*Yeh et al.*, this issue (*b*)] as

$$U = (\bar{K}_{22}'/\bar{K}_{11}' - 1) = \exp[\sigma_f^2(1 - \zeta H)^2/(1 + A\lambda_1)] - 1 \quad (6)$$

The parameters for their Figure 4 are $K_{s2}/K_{s1} = 10$, $\alpha_2/\alpha_1 = 6$, and $D_2/D_1 = \frac{10}{6}$. Assuming the composite medium has a bimodal distribution of conductivity, the variance of log-saturated hydraulic conductivity is evaluated to be $\sigma_f^2 = 1.32$. Since the mean capillary pressure is zero at saturation ($P^r = 2.28$) and the correlation scale is taken to be one half of the average layer thickness ($\lambda_1 = 4.0$ cm), the mean slope $A = 0.254$ cm^{-1} from (5). The knowledge of A and $\alpha_2/\alpha_1 = 6$ yields values of α_1 and α_2, 0.0727 cm^{-1} and 0.436 cm^{-1} respectively. The two-layered deterministic system can be thought of as one with α and $\ln K_s$ perfectly correlated; therefore $\zeta = (\alpha_2 - \alpha_1)/(\ln K_{s2} - \ln K_{s1}) = 0.158$. The $K - \psi$ curves of these two soils should cross at a specific capillary pressure H^* which corresponds to the isotropic condition ($U = 0$), $\zeta H^* = 1$ in (6). Their Figure 4 gives $U = 0$ when $P^r = 0.63$ and from (5) this gives $\zeta = 0.158$, which is identical to the above estimate. Assuming that these estimated parameters are comparable to their case, the anisotropy coefficient as a function of P^r is evaluated implicitly using (5) and (6); the result is illustrated in Figure 5. This figure shows reasonable agreement between the results of the two models. However, the comparison is not unique because some of the required parameters are not given explicitly. For example, the angle of inclination is unknown in their analysis so that the theoretical analysis assumes a small angle. However, the comparison does demonstrate that both models derive the same general behavior for the anisotropy of unsaturated conductivity.

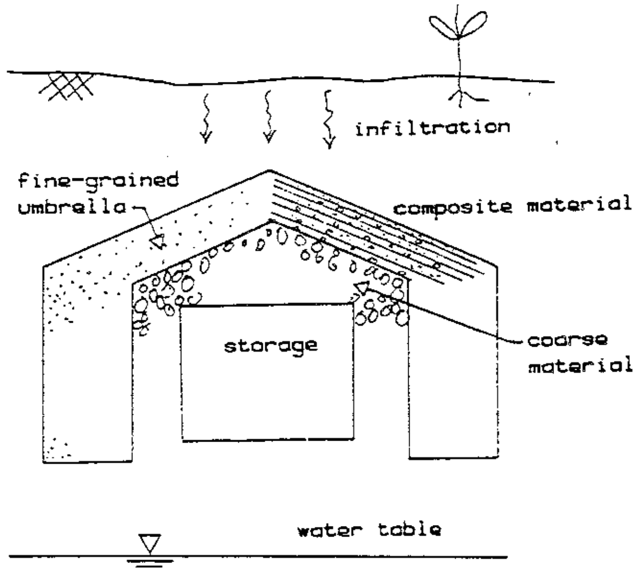

Fig. 6. Configuration of the shallow underground waste storage facility above the water table. The left half illustrates the traditional design and the right half shows the composite material improvement suggested by the stochastic theory.

Practical Implication of the Results

The most interesting result arising from the theoretical analysis is the anisotropy of unsaturated hydraulic conductivity. The unique nature of the anisotropy, which depends on the degree of soil saturation, may have been postulated in the past. However, conclusive results on this aspect of unsaturated conductivity have not been reported in the literature. Such a moisture dependent anisotropy can lead to unexpectedly large horizontal flow components in unsaturated media. The magnitude of the horizontal flow may be important in several practical situations which involve horizontal changes in moisture content.

One problem area in which the lateral flow effect may be significant is that of groundwater recharge from open channels. In the classic approach, seepage from open channels is analyzed by a one-dimensional vertical unsaturated flow approach [*Bouwer*, 1964], but recent studies [*Jeppson and Nelson*, 1970; *Vauclin et al.*, 1979] show that these approaches do not adequately describe the actual flow system in the field due to the presence of strong lateral capillary pressure gradients; multidimensional unsaturated flow analysis is suggested. Moreover, since most soils exhibit bedding, the nonuniformity and stratification of the soils produce a moisture-dependent anisotropy in which the horizontal hydraulic conductivity can be many times the vertical, depending on the saturation of the soils. Thus the vertical flow component may no longer be dominant and a three-dimensional flow situation with large lateral flow components may develop. Such a three-dimensional phenomenon is not adequately described by a classical three-dimensional flow model with isotropic conductivity or anisotropic conductivities of a constant ratio [e.g., *Freeze*, 1971]. The lateral flow resulting from the moisture-dependent anisotropy may restrict vertical movement of water to the groundwater. Consequently, water may be confined to a shallow depth so that the net recharge to groundwater systems could be less than that predicted by the classical approach.

The saturation dependent anisotropy may play an important role in problems of pollutant migration in the vadose zone. According to theoretical results, the horizontal unsaturated hydraulic conductivity of a stratified soil formation could be several orders of magnitude greater than the vertical unsaturated hydraulic conductivity while the vertical conductivity decreases considerably as mean capillary pressure increases. As the soil becomes drier, the horizontal hydraulic conductivity becomes increasingly important relative to the vertical conductivity causing migration of water in the horizontal direction. When pollutants escape from a waste disposal site into a geological formation where interbedded clay and sand are dominant, the plume of pollutants can migrate a substantial horizontal distance in the unsaturated zone before reaching the water table. This phenomenon needs to be considered in the design of waste disposal facilities.

The theoretical results and field and laboratory observations suggested a possible method for wastewater disposal. The use of controlled application rates of wastewater in stratified unsaturated soil formations with low natural moisture content could be an effective way to isolate wastewater. Since migration of pollutants in this type of environment is predominantly in the horizontal direction, wastewater applied on the surface could be confined to the unsaturated zone, thereby reducing the potential for groundwater pollution.

Another practical application of the anisotropy of unsaturated media can be illustrated in the design of geologic environments using the unsaturated hydraulic conductivity properties of soils for waste storage facilities in the shallow subsurface. Under unsaturated conditions, a gravel lens will cause lateral flow in a finer-textured material situated above the gravel. *Corey and Horton* [1969] refer to this phenomenon as the "wick effect"; *Frind et al.* [1977] developed simulations illustrating the potential usefulness of the wick effect in the design of a waste storage facility.

The basic design of the storage facility is illustrated in Figure 6. The waste container situated above the water table is enclosed in a gravel layer with a sloping surface at the top. Finer-grained material overlying the gravel layer forms the wick layer. Due to drastic contrasts in the unsaturated hydraulic conductivity of the two layers, water is confined in the finer-textured material. The sloping interface creates lateral hydraulic gradients in the finer-grained material and forms a protective "umbrella" for the waste container.

A more desirable material for this protective umbrella is an anisotropic medium with a larger conductivity parallel to the layer. In this way, the vertical flow of water into the waste storage area can be further reduced. According to the anisotropy ratio formula derived from the theory, a protective layer having a large conductivity anisotropy can be produced by an appropriate arrangement of different textured materials. To achieve a large anisotropy, (34*b*) of part 2 [*Yeh et al.*, this issue (*b*)] suggests that the material overlying the gravel layer should be perfectly stratified and possess large variances of ln K_s and pore-size distribution parameters. This equation implies that it is more advantageous to use a composite medium consisting of a mixture of layers of fine- and course-textured material than a single uniform homogeneous fine-grained material. If the correlation scale is regarded as the average thickness of the layers, this equation also indicates that the thickness of each layer in the composite medium should be as small as possible.

Saturation-dependent anisotropy also has implications in surface hydrology as discussed by *Zaslavsky and Sinai* [1981*a*, *b*]. Briefly, they found that the anisotropy of unsaturated soils and the slope of the land surface can produce a lateral flow component. This horizontal flow, in turn, causes moisture accumulation in concave parts of the landscape to the point of saturation. This concentration of water in concave areas explains some rainfall-runoff and erosion phenomena that were previously unexplained by classical concepts of vertical infiltration.

SUMMARY AND DISCUSSION

The results of the stochastic theory developed in part 2 [*Yeh et al.*, this issue (*b*)] predict the general behavior of the variability of capillary pressure or moisture content in a field situation. Both theory and observations indicate that the variance of capillary pressure increases with its mean value; i.e., the variation in capillary pressure becomes larger when the soil is dryer.

Moisture or mean capillary pressure dependent anisotropy of the unsaturated hydraulic conductivity is demonstrated by the theoretical model with the hydraulic parameters of Panoche silty clay loam and Maddock sandy loam. The anisotropy ratio of unsaturated hydraulic conductivity for Maddock sandy loam varies strongly with the mean capillary pressure, whereas that of silty clay loam remains practically constant; this is attributed to the fact that Maddock sandy loam has a higher variance of the parameter α reflecting variations in soil texture. Several field observations and laboratory experiments are found to be in qualitative agreement with the moisture-dependent anisotropy concept. Anisotropy ratios calculated by the stochastic model also agree with those from a deterministic model for a two-layered soil.

The importance of the correlation scale in effective hydraulic conductivity is also demonstrated through a direct comparison of the anisotropy ratios derived from both the stochastic result and a direct average method.

Through the examples of recharge, pollutant migration, and rainfall-runoff problems in the large-scale vadose zones, the importance of the new unsaturated hydraulic conductivity anisotropy concept in hydrologic applications is stressed. The theoretical results are applied to study practical problems associated with liquid waste isolation, pollutant migration, and design of capillary barriers for shallow underground waste storage. However, applications of the theoretical results to field situations require information on the statistical properties of the processes, ln K_s and α. These statistical parameters include σ_f^2, σ_a^2, λ_1, λ_2, λ_3, and their autocorrelation functions and cross-covariance function. Information on the statistical properties of saturated hydraulic conductivity is already quite extensive; *Gelhar and Axness* [1983] discuss several possible ways to obtain the estimates of these parameters. Difficulties lie in the determination of the statistical properties of the parameter α. Estimates of these properties of α can be determined only after a large number of $K - \psi$ curves are obtained, but in general, the measurements of $K - \psi$ relationships is time consuming and laborious. While obtaining reasonable estimates of these properties can be a formidable task, the instantaneous profile method [*Neilsen et al.*, 1973] may provide a relatively simple approach to this problem.

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