WATER RESOURCES RESEARCH, VOL. 26, NO. 6, PAGES 1271–1279, JUNE 1990

# Effective Unsaturated Hydraulic Conductivity of Layered Sands

T.-C. JIM YEH AND DONALD J. HARVEY

*Department of Hydrology and Water Resources, The University of Arizona, Tucson*

Accurate estimates of field-scale hydraulic conductivities of unsaturated heterogeneous soils are very difficult to obtain. In the present study, various approaches to determining effective conductivity values for heterogeneous sands are compared with laboratory measurements. The unsaturated hydraulic conductivity, $K(\psi)$, of two homogeneous sands and one layered sand composed of the two homogeneous sands was measured using the steady-state flux control method. The averaged $K(\psi)$ curves of the two homogeneous sands using a direct averaging approach were compared with the measured layered sand $K(\psi)$ data. The result shows that the geometric mean of hydraulic conductivity-suction curves of the coarse and medium sands approximates the measured hydraulic conductivity-suction curve of the layered sand. The observed suction variance and effective hydraulic conductivity were compared with expressions developed from a stochastic theory. The results support the stochastic approach.

## INTRODUCTION

Field soils are inherently heterogeneous. The heterogeneity or spatial variability of soil hydrologic parameters is well documented [*Warrick and Nielson*, 1980]. Hydraulic properties of soils have been shown to be variable in both planes and depths, and the variation of the properties is also spatially correlated [*Russo and Bresler*, 1981; *Vieira et al.*, 1981; *Byers and Stephens*, 1983; *Greenholtz et al.*, 1988].

The importance of considering spatial variability in assessing flow and solute transport in the unsaturated zone has been examined in a number of theoretical works [e.g., *Yeh et al.*, 1985*a*, *b*, *c*; *Mantoglou and Gelhar*, 1987*a*, *b*, *c*]. The fundamental question is how can this heterogeneity best be incorporated in a quantitative description of flow and solute transport in large-scale unsaturated systems. Two different approaches can be identified [*Yeh*, 1989*a*].

The first approach to evaluate flow or solute transport in heterogeneous unsaturated soils is to construct a detailed deterministic model that reflects the actual heterogeneity of the region at all scales of observation. Although there exists a vast body of literature for vertical infiltration in layered soils [e.g., *Tagaki*, 1960; *Zaslavsky*, 1986; *Srinilta et al.*, 1969; *Raats*, 1983; *Sisson*, 1987], most of the studies neglect heterogeneities within layers. The areal variability of soil properties makes this approach even more impractical: characterization of the variability would require enormous amounts of field data and numerical simulations would demand extensive computational resources.

The second approach is to treat the soil as an equivalent homogeneous medium so that the average flow or solute transport characteristics of the system can be predicted. The flow or solute transport model therefore would contain a few parameters that are constant in space. The effects of the heterogeneity can then be treated in the solute transport model as dispersion phenomena [*Gelhar and Axness*, 1983]. This method eliminates the requirement for large amounts of field data and computation but it predicts only the average behavior of the system.

In order to employ the second approach, the effective soil hydrologic parameter values for the equivalent homogeneous medium must be determined. One possible technique for obtaining these effective values is to conduct large-scale field experiments and then apply an inverse method, e.g., by minimizing the differences between the values predicted by the homogeneous equation and observed values. A second technique is to develop a formula that relates the spatial variability of small-scale hydrologic soil parameters to the effective parameters. One example of this technique is the work by *Weir* [1989] in which an analytical expression was derived for the effective hydraulic conductivity of soils with variable saturated hydraulic conductivity. Another example is stochastic analysis.

Using a spectral method *Yeh et al.* [1985*a*, *b*] solved the stochastic partial differential equation describing three-dimensional steady infiltration in porous media of random hydrologic parameters. Their theoretical results derived from the stochastic analysis (hereinafter designated as the stochastic results) show that the variance of soil-water suction increases with the mean suction value. They also derived analytical expressions for the horizontal and vertical effective hydraulic conductivities of heterogeneous soils that depend on the spatial statistics of soil hydrologic properties (e.g., the covariance of the saturated hydraulic conductivity and a pore-size distribution parameter).

Although soil-water pressure data collected by *Yeh et al.* [1986] and *Greenholtz et al.* [1988] support the mean-dependent head variance concept, the formula developed by *Yeh et al.* [1985*a*, *b*] for effective unsaturated hydraulic conductivity remains to be tested.

The results of recent numerical studies of steady infiltration in random porous media by *Ababou and Gelhar* [1988] show promise for the effective hydraulic conductivity formula of *Yeh et al.* [1985*a*, *b*]. However, the cases examined were rather limited. A more comprehensive analysis of vertical infiltration in layered soils by *Yeh* [1989*b*] indicated that while suction variances agree well with those obtained by *Yeh et al.* [1985*a*, *b*], the effective hydraulic conductivity, on the other hand, seems to be larger but is in the same order of magnitude as the result of the stochastic analysis. The agreement in head variance complements the conclusion by *Gutjahr* [1984] in which he stated that the head variance expression from the stochastic approach is exact. However,

Paper number 89WR03638.
0043-1397/90/89WR-03638$05.00

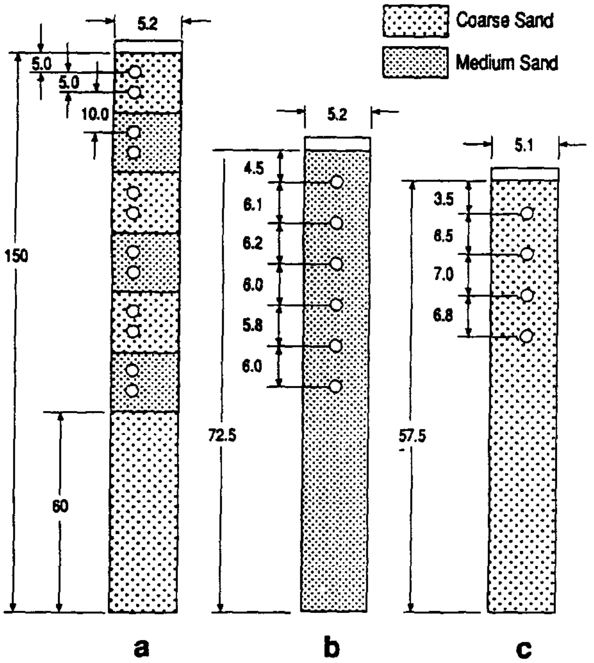

Fig. 1. Diagram of (*a*) layered, (*b*) medium, and (*c*) coarse sand columns (dimensions in centimeters; not to scale). Circles represent tensiometer locations.

the validity of using a exponential extrapolation to obtain the effective unsaturated hydraulic conductivity in the spectral analysis was questioned by *Poley* [1988]. Numerical experiments by *Bosch and Yeh* [1990] demonstrated that the effective hydraulic conductivity can be used to simulate the mean suction profile in layered soils under one-dimensional transient flow conditions.

To fully assess the usefulness of the stochastic results, many field experiments are necessary. However, a logical step toward the validation is to test the stochastic results against data collected from controlled laboratory experiments. This is the major objective of this paper. To test the stochastic results using three-dimensional laboratory experiments is a difficult task. In this paper laboratory experiments of vertical infiltration in soil columns which test the mean-dependent head variance and the effective hydraulic conductivity concepts were described.

## MATERIALS AND METHODS

The study consisted of a series of steady-state infiltration experiments alternately conducted on three different sand columns. The first contained a coarse sand, the second a medium sand, and the last alternating layers of the coarse and medium sands (Figure 1). The layered column consisted of alternating layers (approximately 15 cm thick) of coarse and medium sands starting with a 60 cm long coarse layer at the bottom of the column and ending with a coarse layer at the top. To maintain a roughly constant bulk density throughout the column the sand was firmly tamped with a closely fitting plunger after each addition of about 10 cm of sand poured through a funnel.

The dry bulk density of each sand was determined by *Mathieu* [1989]. Ten samples of the coarse sand and eight samples of the medium sand were packed under different stresses equivalent to the weight of the sand columns. The mean and standard deviation of the dry bulk density of the medium sand are 1.46 and 0.042 g/cm$^3$. The mean and standard deviation values for the coarse sand are 1.50 and 0.032 g/cm$^3$.

TABLE 1. Dry Bulk Densities and Porosities of the Sands

| Sand Column | Bulk Density, g/cm$^3$ | Porosity |
|---|---|---|
| Coarse | 1.51 | 0.430 |
| Medium | 1.45 | 0.454 |
| Layered | | |
| Layer 1 (coarse) | 1.46 | 0.449 |
| Layer 2 (medium) | 1.46 | 0.449 |
| Layer 3 (coarse) | 1.53 | 0.423 |
| Layer 4 (medium) | 1.50 | 0.434 |
| Layer 5 (coarse) | 1.56 | 0.411 |

Assuming a particle density of 2.65 g/cm$^3$, the mean and standard deviation of porosity values for the medium sand are 0.449 and 0.016, while the coarse sand values are 0.435 and 0.012, respectively. These bulk density and porosity values compare favorably with dry bulk densities and porosities determined for the columns used in this study (Table 1). The grain size distributions for these sands are shown in Figure 2.

Wetting and drying retention curves for the sands were also obtained by *Mathieu* [1989] using a hanging water column method, and are shown in Figure 3. Three replicate samples each of the medium and the coarse sands were analyzed. The mean and standard deviation of residual moisture contents (moisture at a suction of 160 cm) are: for the coarse sand, 0.047 and 0.012; and for the medium sand 0.067 and 0.015, respectively. The mean and standard deviation of saturated moisture contents are: for the coarse sand 0.446 and 0.005; and for the medium sand 0.44 and 0.003. These retention curves were assumed to be representative retention curves of the coarse and medium sand columns.

Unsaturated hydraulic conductivity data, $K(\psi)$ where $\psi$ denotes suction (a positive value under unsaturated condi-

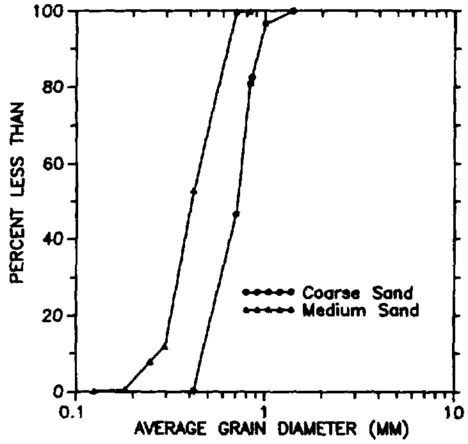

Fig. 2. Grain size distributions of the coarse and medium sands used in the experiment.

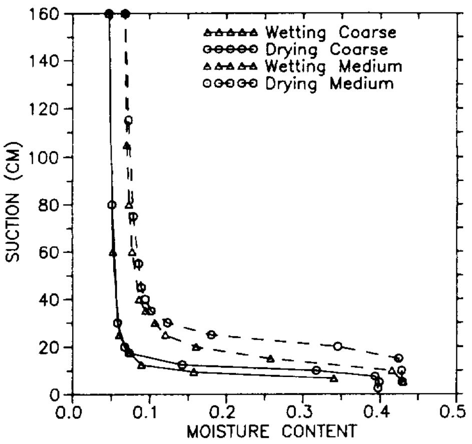

Fig. 3. Drying and wetting water retention curves of the coarse and medium sands.

tions), were obtained for all three sand columns using the long-column version of the steady-state flux control method [*Klute and Dirksen*, 1986]. The experimental setup for this method in the study is illustrated in Figure 4. It includes a soil column, a tensiometer-manometer system, a multichannel syringe pump (soil measurement system), and a recycle tank. Tensiometers with water manometers were used to measure the suction in the sand: four, six, and twelve tensiometers were used for the coarse, medium, and layered sand columns, respectively. Each tensiometer-manometer consisted of a 0.64 cm O.D. and 2.9 cm long porous ceramic cup (high flow, 0.5 bar, Soil Moisture Corporation) into which was inserted and glued with epoxy adhesive two meters of 0.32 cm O.D. tygon tubing. The tensiometer-manometer systems were checked for air leaks before they were installed at various heights of the upper portion of the soil columns (Figure 1).

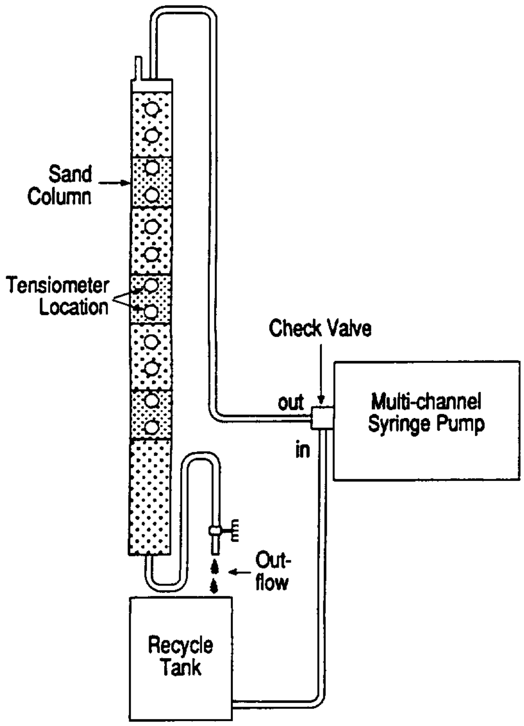

Fig. 4. Experimental setup for this study.

This steady-state flux control method uses a constant flux of water into the upper end and a water table at the bottom. If the soil column is initially saturated and a constant water flux less than the saturated hydraulic conductivity of the sand is applied, the column will drain to a condition of steady-state downward flow. Upon reaching this steady-state flux condition the upper portion of the column, if the soil is uniform, will achieve a state of relatively constant suction or moisture content. A unit hydraulic gradient will thereby be established throughout the upper region.

However, since sand columns are never perfectly uniform, variation in suctions always exists throughout the "uniformly packed" sand column. For the layered column, variations in suction are even more significant. Therefore, a unit mean gradient approach suggested by *Yeh* [1989*b*] is used. That is, in spite of the variation in the suction profile in the layered sand, as with the coarse and medium sands, the mean suction value (the averaged suction values observed at all the tensiometers) was considered to represent the unit gradient suction value for purposes of determining the $K$-$\psi$ function. Hereafter, $K(\psi)$ refers to the effective hydraulic conductivity as a function of mean suction, $\psi$ [*Yeh*, 1989*b*].

Assuming Darcy's law holds for unsaturated flow, the hydraulic conductivity, $K(\psi)$, numerically equals the volumetric flux, $q$, under unit gradient conditions. By measuring suctions at various heights throughout the unit mean gradient region, the $K(\psi)$ relationship can be measured. Starting at saturation (achieved by a vacuum wetting procedure) and proceeding through a series of progressively decreasing flow rates, the draining or drying $K(\psi)$ function can be obtained. The wetting $K(\psi)$ function was acquired by starting at the lowest flux or conductivity value of interest in a column that was dryed by drawing air through it until the sand was visibly dry and then establishing a series of progressively increasing flow rates. Because of the difficulty in controlling flow rates near saturation, no measurement of the wetting function was conducted near saturation.

In general, it takes a much longer time to measure the wetting function because of the very low initial conductivity of the sand and the low starting flow rates. For this reason, measurement of unsaturated hydraulic conductivities generally do not include the wetting sequence [*Klute and Dirksen*, 1986]. As was observed in this study, measurements on heterogeneous sands can take much longer than on homogeneous sands. This apparently is due to the individual layers reaching a steady-state condition in stepwise fashion starting with the uppermost layer. Since subsequent suction changes in the lower layers affect flow conditions in overlying layers the upper layers' initial equilibrium conditions become altered. This cycle was observed to repeat itself throughout the column many times until the incremental suction head changes within each layer became negligible.

## ANALYSIS OF RESULTS

### *Analysis of Measured Hydraulic Conductivity Data*

Saturated hydraulic conductivity values of the coarse and medium sand columns were determined using a constant head method to be 0.1126 and 0.0905 cm/sec with standard errors of 0.0029 and 0.0016 cm/sec, respectively. For the

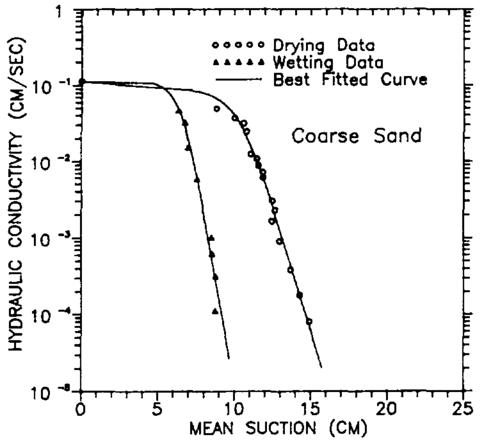

Fig. 5. Drying and wetting hydraulic conductivity-suction data of the coarse sand.

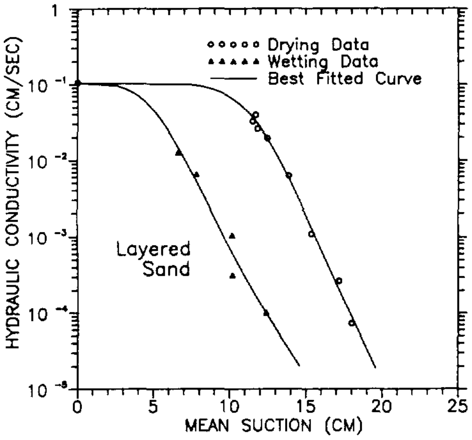

Fig. 7. Drying and wetting hydraulic conductivity-suction data of the layered sand.

layered column, the saturated hydraulic conductivity was 0.1014 cm/sec which is close to the harmonic mean of the coarse and medium sand values. Experimental wetting and drying $K$-$\psi$ data of the coarse, medium and layered sand columns were measured using the method discussed previously and are shown in Figures 5, 6, and 7, respectively.

In order to describe the value of $K(\psi)$ for all values of suction, $\psi$, two mathematical models were used. A model developed by *van Genuchten* [1978] based on *Mualem*'s [1976] theory for predicting the $K(\psi)$ function from soil-water retention data $\theta(\psi)$ is as follows:

$$K(\psi) = \{1 - (\alpha\psi)^{n-1}[1 + (\alpha\psi)^n]^{-m}\}^2/[1 + (\alpha\psi)^n]^{m/2} \qquad (1)$$

where $m = 1 - 1/n$. The moisture content-suction relationship is given by:

$$\theta = (\theta_s - \theta_r)[1 + (\alpha\psi)^n]^{-m} + \theta_r \qquad (2)$$

where $\theta$ is the moisture content, $\theta_r$ is the residual moisture content, and $\theta_s$ is the saturated moisture content. $\alpha$ and $n$ are parameters to be fitted to moisture retention data.

A non-linear least-squares optimization program [*van Genuchten*, 1978] was used to find the best-fit values of $\alpha$ and $n$ in (1), using the hydraulic conductivity data only. Table 2 contains the best-fit values of $\alpha$ and $n$, their standard errors of estimate, and their $r^2$, square of correlation coefficient values.

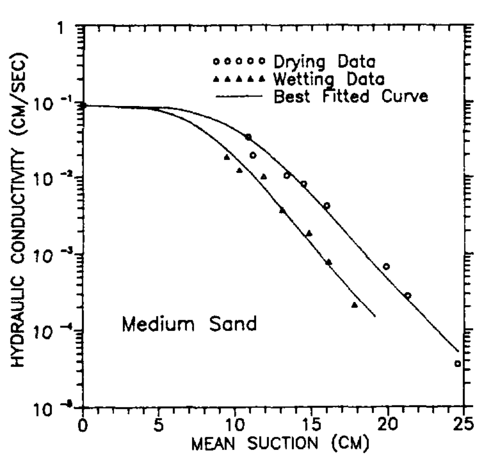

Fig. 6. Drying and wetting hydraulic conductivity-suction data of the medium sand.

As a check on the ability of the van Genuchten model to predict the $K(\psi)$ function from water retention data for these sands, $\alpha$ and $n$ values were also determined from retention data alone (Table 2). Significantly different $\alpha$ and $n$ values resulted from the optimization process using the two different types of data. A comparison of the retention- and conductivity-based wetting hydraulic conductivity curves for the coarse sand illustrates the difference (Figure 8). The discrepancy may be attributed to the fact that different samples were used to measure the retention curves. However, as mentioned previously, the bulk densities of the samples compare favorably with those determined from the sand columns. This result may indicate that the parameters obtained from retention curves are not suitable for predicting the unsaturated hydraulic conductivity.

The exponential model proposed by *Gardner* [1958] is as follows:

$$K(\psi) = K_s \exp(-\beta\psi) \qquad (3)$$

where $K_s$ is the saturated hydraulic conductivity, $\beta$ is the pore-size distribution parameter which characterizes the rate of reduction in hydraulic conductivity with increasing suction $\psi$. Equation (3) can be linearized and rewritten as:

$$\ln K = \ln K_s - \beta\psi \qquad (4)$$

where ln is the natural logarithm. The exponential model is used here because it is the model upon which the stochastic expressions developed by *Yeh et al.* [1985*b*] are based, the experimental verification of these expressions being a primary objective of this study. The $\beta$ and ln $K_s$ values were obtained by fitting (4) to the observed data and are listed in Table 3. However, the exponential model does not account for $K$ values at suction lower than the air-entry value. The best-fit intercept, therefore, is an artificially high log saturated conductivity. The exponential model is not valid below the observed air-entry suction value for the sand. According to Figure 5 and 6, the air-entry occurs at a suction of about 10 cm for the coarse sand and about 5 cm in the medium sand. These values are somewhat counterintuitive. However, the actual air-entry values are unknown because of difficulties in controlling the flow rate near saturation.

TABLE 2. Parameters of van Genuchten's Unsaturated Hydraulic Conductivity Model for the Sands

| | Retention-Based | Conductivity-Based |
|---|---|---|
| Drying | $n$ = 6.79 (0.594) | $n$ = 9.30 (0.196) |
| Coarse | $\alpha$ = 0.0956 (00154) | $\alpha$ = 0.092 (0.00034) |
| | $r^2$ = 0.986 | $r^2$ = 0.989 |
| Drying | $n$ = 7.18 (0.372) | $n$ = 4.72 (0.924) |
| Medium | $\alpha$ = 0.045 (0.00047) | $\alpha$ = 0.074 (0.00057) |
| | $r^2$ = 0.994 | $r^2$ = 0.990 |
| Wetting | $n$ = 5.28 (0.273) | $n$ = 10.16 (0.532) |
| Coarse | $\alpha$ = 0.134 (0.0021) | $\alpha$ = 0.143 (0.000137) |
| | $r^2$ = 0.986 | $r^2$ = 0.968 |
| Wetting | $n$ = 4.53 (0.069) | $n$ = 4.27 (0.115) |
| Medium | $\alpha$ = 0.069 (0.00107) | $\alpha$ = 0.0913 (0.00074) |
| | $r^2$ = 0.993 | $r^2$ = 0.984 |
| Drying | ⋯ | $n$ = 7.015 (1.93) |
| Layered | ⋯ | $\alpha$ = 0.0781 (0.00215) |
| | | $r^2$ = 0.957 |
| Wetting | ⋯ | $n$ = 4.288 (0.460) |
| Layered | ⋯ | $\alpha$ = 0.149 (0.0066) |
| | | $r^2$ = 0.976 |

Standard errors in parentheses; $r^2$, square of correlation coefficient.

A finite element program, UNSATID, using the chord-slope iteration scheme to solve the one-dimensional Richards' equation was developed based on the program by *Khaleel and Yeh* [1985]. It was employed to simulate suction profiles in the layered sand column under different infiltration rates. The soil hydraulic parameters used were the measured $K$-$\psi$ of the coarse and medium sand columns. Transient simulations with a constant flux upper boundary condition and a water table condition at the lower boundary were carried out. Figures 9 and 10 show the comparison of the simulated and observed steady-state suction profiles during drying and wetting cycles, respectively, in the layered sand column. The simulated profiles approximate the experimental data. Deviations may be attributed partially to the fact that the measured hydraulic properties of the coarse and medium sand columns do not represent those of the sands in the layered sand column. They may be also caused by the fact that the size of the tensiometer is not large enough to obtain a representative average suction value.

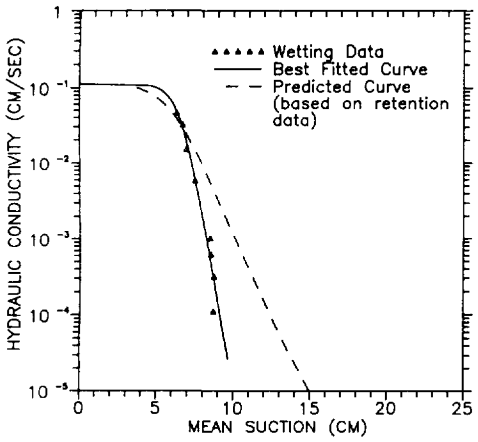

Fig. 8. A comparison of the measured hydraulic conductivity-suction data and the data derived theoretically based on the water retention data.

### *Suction Variance*

Figures 9 and 10 also illustrate that the difference in suction values in the coarse and medium layers of the layered sand column increases as the flux decreases. In other words, the drier the soil, the greater the variation in head in the layered soil column. This result qualitatively supports the mean-dependent head variance concept by *Yeh et al.* [1985*b*]. To provide a quantitative comparison, we first assume ergodicity applies and then calculate the mean and variance of experimental suction data. The mean suction value represents the arithmetic average of all suction values observed throughout the layered sand column under a given infiltration rate. The variance denotes the deviation of observed suction values from the mean. Means and variances of the suction profiles under various infiltration rates were then compared to a stochastic model by *Yeh et al.* [1985*b*].

Using the exponential hydraulic conductivity model, *Yeh et al.* [1985*b*] developed an expression for the head variance. Although the expression is developed for field-scale analyses, it is also suitable for the layered sand column. Since the layered soil column can be treated as the case where ln $K_s$ and $\beta$ are perfectly correlated, the suction variance at a unit mean gradient condition is given by [*Yeh et al.*, 1985*a*, *b*, *c*; *Yeh*, 1989*b*]:

$$\sigma_\psi^2 = \frac{\sigma_f^2 l^2 (1 - H\xi)^2}{Bl(1 + Bl)} \tag{5}$$

where $\sigma_\psi^2$ is the suction variance, $\sigma_f^2$ is the variance of ln $K_s$, $H$ denotes the mean suction, $B$ is the mean $\beta$, $l$ represents the integral scale, and $\xi^2 = \sigma_\beta^2/\sigma_f^2$ where $\sigma_\beta^2$ is the variance of $\beta$. The calculated means and variances of the parameter values for the layered sand are listed in Table 4. It should be noted that (5) was derived using an exponential covariance function for both ln $K_s$ and $\beta$ parameters, which does not represent the spatial correlation structure of the layered sand. However, *Yeh et al.* [1985*a*] showed that the head variance is independent of the form of covariance function (see *Yeh et al.*, 1985*a*, Figure 2) as the value of B$l$ becomes

TABLE 3. Exponential Model Parameters for the Sands

| | $\ln K_s$ | $\beta$, cm$^{-1}$ | $r^2$ |
|---|---|---|---|
| Drying, coarse | 8.666 (0.969) | 1.184 (0.0802) | 0.940 |
| Drying, medium | 1.781 (0.411) | 0.4716 (0.024) | 0.985 |
| Wetting, coarse | 11.21 (1.65) | 2.206 (0.212) | 0.948 |
| Wetting, medium | 1.202 (0.570) | 0.5225 (0.417) | 0.969 |
| Drying, layered | 7.512 (0.544) | 0.9293 (0.038) | 0.990 |
| Wetting, layered | 1.480 (1.116) | 0.8675 (0.116) | 0.949 |

Standard errors in parentheses; $r^2$, square of correlation coefficient.

greater than 1. Inasmuch as the value of *Bl* of the layered sand is much greater than 1 (10 for wetting, and 6 for drying), in the following comparison between the theory and the data, the effect of the covariance function was assumed negligible. That is, the layered sand was assumed to possess an exponential covariance function, and the correlation length was assumed to be half the thickness of the layer in the column.

Because Figures 9 and 10 show that a unit mean gradient exists at the portion of the sand columns where the tensiometers were installed, the means and variances of the observed suction values were estimated from suction readings obtained from all the tensiometers. The theoretical suction variance as a function of mean suction for the layered sand is plotted with the estimated means and variances of the suction values from the experimental data in Figure 11. The theoretical head variance curve agrees very well with the wetting data. It exhibits the same trend as the wetting and drying data, regardless of the exponential covariance assumption. Furthermore, as illustrated in Figure 11, the suction variance is greater in the case of wetting than that in the drying. This result also agrees with that of *Mantoglou and Gelhar* [1987*b*].

### *Effective Hydraulic Conductivity*

*Direct averaging approach.* It is not clear that the hydrologic properties of individual homogeneous soils within a large-scale heterogeneous soil are additive, i.e., whether or not the properties of the whole represent some kind of a mean of the properties of its component parts. Arguments have been made that additivity is not possible under transient flow conditions [*Philip*, 1980] or that it is not realistic since it does not address the stochastic nature of heterogeneous soils [*Yeh et al.* 1985*c*]. In spite of these arguments, a direct averaging approach [*Mualem*, 1984; *Yeh et al.*, 1985*c*] may still prove to be of some use in estimating effective parameters such as hydraulic conductivity. By direct averaging, we mean averaging the hydraulic conductivity values of the homogeneous coarse and medium sand columns at a fixed suction value and repeating the process for a number of different suctions over the range of interest. This procedure results in an effective $K(\psi)$ function. Three types of means for direct averaging were used, namely: arithmetic mean, geometric mean, and harmonic mean.

These mean curves are plotted with the layer curves in Figures 12 and 13. The geometric mean closely approximates the experimental data, especially the drying experimental data.

To see if the effective curve can be approximated by averaging the parameters in (1) directly, the arithmetic, geometric, and harmonic means of the coarse and medium sand parameters ($K_s$, $n$, and $\alpha$) were calculated. These mean parameter values along with $K_s$, $n$, and $\alpha$ values of the coarse, medium, and layered sands are listed in Table 5. Although not conclusive the arithmetic mean closely approximates the drying layered sand parameter values while none

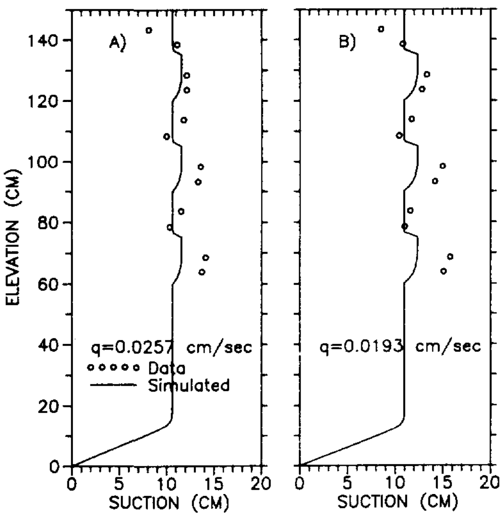

Fig. 9. Comparisons of the simulated and observed suction profiles under two different steady-state flow (drying) in the layered sand column, (*a*) $q$ = 0.0257 cm/sec and (*b*) $q$ = 0.0193 cm/sec.

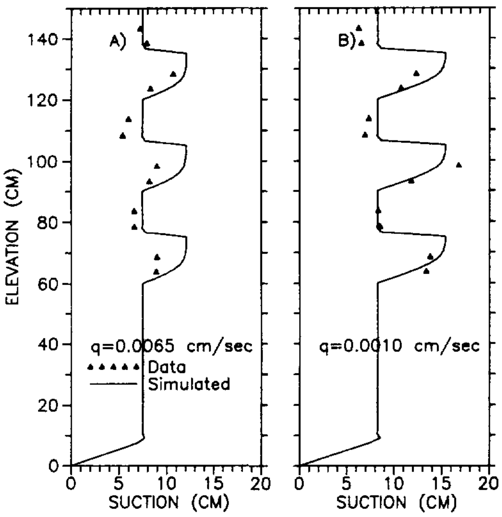

Fig. 10. Comparisons of the simulated and observed suction profiles under two different steady-state flow (wetting) in the layered sand column, (*a*) $q$ = 0.0065 cm/sec and (*b*) $q$ = 0.0010 cm/sec.

TABLE 4. Stochastic Values for the Layered Sand

| Parameter | Wetting | Drying |
|---|---|---|
| $\sigma_f^2$ | 25.03 | 11.852 |
| $\sigma_\beta^2$ | 0.7089 | 0.1270 |
| $B$ | 1.3645 | 0.8279 |
| $F$ | 6.2049 | 5.2240 |
| $l$ | 7.5 cm | 7.5 cm |

of the means based on wetting parameters can be considered a close match.

*Formula based on the stochastic results.* The expression for the effective unsaturated hydraulic conductivity perpendicular to the layer derived from the stochastic approach of *Yeh et al.* [1985*a*, *b*, *c*] is given by [*Yeh*, 1989*b*]

$$\hat{K}_e(H) = \hat{K}_s \exp(-\hat{\beta} H) \tag{6}$$

for perfectly correlated ln $K_s$ and $\beta$; where $\hat{K}_e$ is the effective unsaturated hydraulic conductivity, $H$ is the mean suction, and

$$\hat{K}_s = \exp\left[F - \frac{\sigma_f^2}{2(1+Bl)}\right] \tag{7}$$

and

$$\hat{\beta} = B + \frac{\sigma_f^2(H^2\xi^2 - 2H\xi) - 2\sigma_\beta^2(H\xi - 1)l/\xi}{2(1+Bl)H} \tag{8}$$

where $\hat{K}_s$ represents the effective conductivity at $H = 0$, $\hat{\beta}$ denotes the effective $\beta$ parameter, and $F$ is the expected value of ln $K_s$.

Using (7) and (8), (6) can also be rewritten as

$$\hat{K}_e(H) = K_g \exp\left[-\frac{\sigma_f^2}{2(1+Bl)} - \frac{\sigma_f^2(H^2\xi^2 - 2H\xi) - 2\sigma_\beta^2(H\xi - 1)l/\xi}{2(1+Bl)H}\right] \tag{9}$$

where $K_g = \exp(F - BH)$, representing the geometric mean of $K$-$\psi$ curves of different soils. The exponent represents the effect of variations in ln $K_s$ and $\beta$ on the effective hydraulic conductivity.

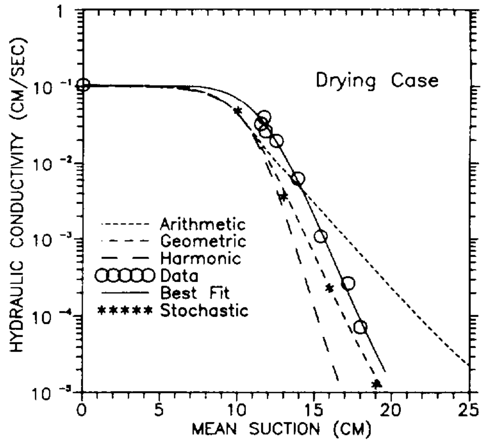

Fig. 12. Comparisons of measured hydraulic conductivity as a function of suction during drying experiments and those from direct averaging and the stochastic results.

The experimental values for the variables in (6)–(8) are listed in Table 4. Comparisons of the measured unsaturated hydraulic conductivity curve of the layered sand and the unsaturated hydraulic conductivity curve resulting from (6)–(8) are plotted in Figures 12 and 13. Stochastic results agree with the geometric-mean curves, indicating the effect of variations in ln $K_s$ and $\beta$ on effective hydraulic unsaturated conductivity is not significant for this layered sand column at the range of suctions examined. Again, the agreement with the drying data is satisfactory but for the wetting curve, the results are less favorable.

## SUMMARY AND CONCLUSIONS

In this study the unsaturated hydraulic conductivity of coarse, medium, and layered sands was measured. The usefulness of various averaging techniques in estimating the effective hydraulic conductivity of unsaturated layered sands based on the knowledge of the hydraulic conductivity of individual layers was examined. The theoretical expressions derived by *Yeh et al.* [1985*a*, *b*, *c*] for suction variance,

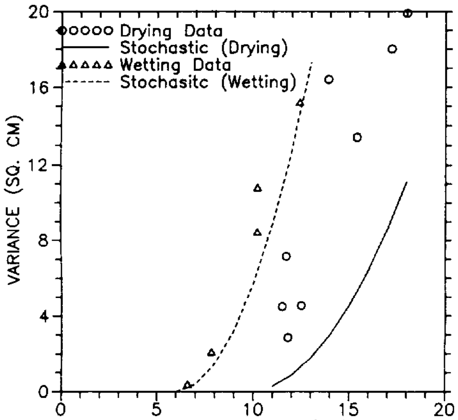

Fig. 11. Comparisons of observed and calculated suction variances in the layered sand column during wetting and drying experiments.

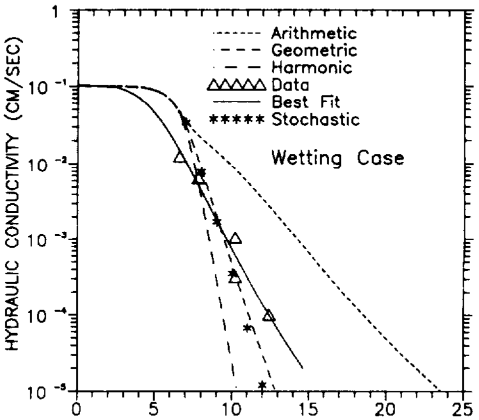

Fig. 13. Comparisons of measured hydraulic conductivity as a function of suction during wetting experiments and those from direct averaging and the stochastic results.

TABLE 5. Van Genuchten Model Parameters and Various Means of the Parameters

| | $K_s$, cm/sec | $n$ | $\alpha$, cm$^{-1}$ |
|---|---|---|---|
| | *Drying* | | |
| Coarse | 0.1126 | 9.30 | 0.091 |
| Medium | 0.0905 | 4.72 | 0.074 |
| Layered | 0.1014 | 7.015 | 0.0781 |
| Arithmetic mean | 0.1016 | 7.010 | 0.0825 |
| Geometric mean | 0.1009 | 6.625 | 0.0821 |
| Harmonic mean | 0.1003 | 6.262 | 0.0816 |
| | *Wetting* | | |
| Coarse | 0.1126 | 10.16 | 0.143 |
| Medium | 0.0905 | 4.27 | 0.0913 |
| Layered | 0.1014 | 4.288 | 0.149 |
| Arithmetic mean | 0.1016 | 7.215 | 0.1172 |

and effective hydraulic conductivity were tested. The suction variance calculated from (5) agrees with the experimental data.

The effective hydraulic conductivity of the layered sand column was measured by employing a unit mean gradient approach suggested by *Yeh* [1989*b*]. Using the measured hydraulic conductivity-suction relationships of medium and coarse sands, different direct averaging techniques were adopted to derive the effective hydraulic conductivity of the layered sand. Results show that the geometric mean of the conductivity values at each suction fits the experimental layered data better than the arithmetic and harmonic means, but none of these averages provides a good estimate of the wetting hydraulic conductivity data of the layered sand.

Arithmetic means of $K_s$, $\alpha$, and $n$ parameter values of medium and coarse sands agree with these parameter values of the drying hydraulic conductivity curve of the layered sand. This technique, however, does not provide a satisfactory result for the wetting curve.

The curve for effective hydraulic conductivity calculated from the stochastic formula fit the experimental drying data fairly well. Calculated conductivities are within one order of magnitude of the experimental data in the range of suctions measured in the column (5 to 20 cm). The calculated $\hat{K}_e$ curves for the drying data, while underestimating the measured $\hat{K}_e$ values, matched the slope of the experimental ln $K$-$\psi$ curve very well.

Overall, the experimental data support the stochastic results of *Yeh et al.* [1985*a*, *b*, *c*]. In general, the stochastic expressions reflect the trend of the experimental data. Although the theoretical curves did not closely fit the wetting effective hydraulic conductivity data, the stochastic approach holds promise as a useful technique in assessing large-scale unsaturated flow and transport. Clearly, additional laboratory and field measurements on more complex soils are called for to better assess the applicability of the stochastic approach.

*Acknowledgments.* Financial support for this research was provided under the Small Grant Program by the University of Arizona Foundation and the Office of the Vice President for Research.

## REFERENCES

Ababou, R., and L. W. Gelhar, Three-dimensional groundwater flow in random media, *M.I.T. Rep. 318*, 883 pp., Cambridge, Mass., 1988.

Bosch, D. D., and T.-C. J. Yeh, Effective hydraulic conductivity for computing one-dimensional flow in heterogeneous porous media, *Trans. ASAE*, in press, 1990.

Byers, E., and D. B. Stephens, Statistical and stochastic analysis of hydraulic conductivity and particle size in a fluvial sand, *Soil Sci. Soc. Am. J.*, *47*, 1072–1080, 1983.

Gardner, W. R., Some steady-state solutions of the unsaturated moisture flow equation with application to evaporation from a water table, *Soil Sci.*, *85*(4), 1958.

Gelhar, L. W., and C. L. Axness, Three-dimensional stochastic analysis of macrodispersion, *Water Resour. Res.*, *19*(1), 161–180, 1983.

Greenholtz, D. E., T.-C. J. Yeh, M. S. B. Nash, and P. J. Wierenga, Geostatistical analysis of soil hydrologic properties in a field plot, *J. Contam. Hydrol.*, *3*(2–4), 227–250, 1988.

Gutjahr, A., Stochastic models of subsurface flow: Log linearized gaussian models are "exact" for covariances, *Water Resour. Res.*, *20*(12), 1909–1912, 1984.

Khaleel, R., and T.-C. J. Yeh, A Galerkin finite element program for simulating unsaturated flow in porous media, *Ground Water*, *23*(1), 1985.

Klute, A., and C. Dirksen, Hydraulic conductivity and diffusivity: Laboratory methods, in *Methods of Soil Analysis, Part I., Agronomy 9*, edited by A. Klute pp. 687–734, Am. Soc. Agron., Madison, Wis., 1986.

Mantoglou, A., and L. W. Gelhar, Stochastic modeling of large scale transient unsaturated flow systems, *Water Resour. Res.*, *23*(1), 37–46, 1987*a*.

Mantoglou, A., and L. W. Gelhar, Capillary tension head variance, mean soil moisture content, and effective specific soil moisture capacity of transient unsaturated flow in stratified soils, *Water Resour. Res.*, *23*(1), 47–56, 1987*b*.

Mantoglou, A., and L. W. Gelhar, Effective hydraulic conductivities of transient unsaturated flow in stratified soils, *Water Resour. Res.*, *23*(1), 57–68, 1987*c*.

Mathieu, J. T., Two-dimensional water flow through stratified unsaturated porous media: Laboratory sand-box experiments, M.S. thesis, Univ. of Arizona, Tucson, 1989.

Mualem, Y., A new model for predicting the hydraulic conductivity of unsaturated porous media, *Water Resour. Res.*, *12*(3), 513–522, 1976.

Mualem, Y., Anisotropy of unsaturated soils, *Soil Sci. Soc. Am. J.*, *48*, 505–509, 1984.

Philip, J. R., Field heterogeneity: Some basic issues, *Water Resour. Res.*, *16*(2), 443–448, 1980.

Poley, A. D., Effective permeability and dispersion in locally heterogeneous aquifers, *Water Resour. Res.*, *24*(11), 1921–1926, 1988.

Raats, P. A. C., Implications of some analytical solutions for drainage of soil-water, *Agric. Water Manage.*, *6*, 161–175, 1983.

Russo, D., and E. Bresler, Soil hydraulic properties as stochastic processes: 1-D analysis of field spatial variability, *Soil Sci. Soc. Amer. J.*, *45*, 699–704, 1981.

Sisson, J. B., Drainage from layered field soils: Fixed gradient models, *Water Resour. Res.*, *23*(11), 2071–2075, 1987.

Srinilta, S., D. R. Nielsen, and D. Kirkham, Steady flow of water through a two-layer soil, *Water Resour. Res.*, *5*(5), 1053–1063, 1969.

Tagaki, S., Analysis of the vertical downward flow of water through a two-layer soil, *Soil Sci.*, *90*, 98–103, 1960.

van Genuchten, M. T., Calculating the unsaturated hydraulic conductivity with a new close-form analytical model, *Res. Rep. 78-WR-08*, Water Res. Program, Civil Eng. Dep., Princeton Univ., 1978.

Vieira, S. R., D. R. Nielsen, and J. W. Biggar, Spatial variability of field measured infiltration rates, *Soil Sci. Soc. Am. J.*, *45*, 1040–1048, 1981.

Warrick, A. W., and D. R. Nielsen, Spatial variability of soil physical properties in the field, in *Applications of Soil Physics* edited by D. Hillel, chap. 13, pp. 319–344, Academic, San Diego, Calif., 1980.

Weir, G. J., Unsaturated vertical flows above a water table, *Water Resour. Res.*, *25*(1), 55–60, 1989.

Yeh, T.-C. J., A review of the scale problem and applications of stochastic methods to determine groundwater travel time and path, *Tech. Rep. HWR 89-010*, pp. 199, Dep. of Hydrol. and Water Resour., University of Arizona, Tucson, 1989*a*.

Yeh, T.-C. J., One-dimensional steady-state infiltration in heterogeneous soils, *Water Resour. Res.*, *25*(10), 2149–2158, 1989*b*.

Yeh, T.-C. J., L. W. Gelhar, and A. L. Gutjahr, Stochastic analysis of unsaturated flow in heterogeneous soils, 1, Statistically isotropic media, *Water Resour. Res.*, *21*(4), 447–456, 1985*a*.

Yeh, T.-C. J., L. W. Gelhar, and A. L. Gutjahr, Stochastic analysis of unsaturated flow in heterogeneous soils, 2, Statistically anisotropic media with variable $\alpha$, *Water Resour. Res.*, *21*(4), 457–464, 1985*b*.

Yeh, T.-C. J., L. W. Gelhar, and A. L. Gutjahr, Stochastic analysis of unsaturated flow in heterogeneous soils, 3, Observations and applications, *Water Resour. Res.*, *21*(4), 465–471, 1985*c*.

Yeh, T.-C. J., L. W. Gelhar, and P. J. Wierenga, Observations of spatial variability of soil-water pressure in a field soil, *Soil Sci.*, *142*(1), 7–12, 1986.

Zaslavsky, D., Theory of unsaturated flow into a non-uniform soil profile, *Soil Sci.*, *97*, 400–410, 1964.

---

D. J. Harvey and T.-C. J. Yeh, Department of Hydrology and Water Resources, University of Arizona, Tucson, AZ 85721.

(Received August 15, 1989;
revised November 27, 1989;
accepted December 7, 1989.)