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A gray system model for studying the response to climatic change: The Liulin karst springs, China

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Summary Gray system theory uses a black-gray-white color spectrum to describe a complex system whose characteristics are only partially known or known with uncertainty. In this study, we use gray system theory to investigate the relation between precipitation and spring flows in a karst region in China. The gray incidence analysis was applied to the Liulin Springs, Shanxi Province, China to analyze the time-lag between spring flow and precipitation. The results showed that the average groundwater residence time at Liulin Spings is about 4 years. The gray system GM(1,2) model was subsequently used as a predictive tool for spring discharge. It was found that model predictions are in agreement with observed data. This study also shows that the discharge of the Liulin Springs primarily responds to climate change; anthropogenic impacts are secondary. The continuous decline of water level in the karst aquifer and waning of spring discharges in semi-arid regions of China might be largely a response of the groundwater system to the decline in regional precipitation over the past two decades.

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Introduction

Global environmental change is a major public concern. Climate trend analyses indicate that global mean air temperature has increased by about 0.3–0.6 °C since the 19th

century (Nicholls et al., 1996). Yang (1999) estimated that the average global air temperature could rise by 1.2 °C from the present time to 2050, and he forecasted that during this same period, in eastern and southeastern China, the average air temperature could rise by 1.0–1.5 °C. In China's entire northern region, the increase in temperature could reach 1.5–2.0 °C by 2050. The direct effect of a global rise in temperature is the redistribution of surface water resources on the earth (Waggoner, 1990). Indirectly, climate

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change may also have effects on land management practices, land use and agricultural practices. For example, irrigation water extracted from aquifers could also alter hydrological systems (Eheart and Tornil, 1999; Loáiciga et al., 2000). Therefore, climate change is of primary importance to hydrological systems and water resource management. Many researchers have been exploring this issue in many other countries throughout the world, as well as in China (e.g., Yang, 1999).

Regional karst aquifers are very vulnerable to climate changes, especially in semi-arid regions. In China, 9.5% of the landmass is karst terrain, which is generally concentrated in two regions. One of the regions encompasses Shanxi Plateau and its neighboring provinces, which covers an area about 470,000 km² and is located in a semi-arid climate zone. The other karst region is located in the southwest region of China, about 500,000 km², and it is in a humid climate zone (Yuan, 1994). Karstification of the regions is highly influenced by precipitation and terrain, which can cause large differences in karst spring flow between different regions. In semi-arid areas of China, most carbonate aquifers are overlain by thick Permian carboniferous-sandstone and shale, and Quaternary sediments. Karstification in these areas is generally not very developed, in contrast to the karst systems in humid, southwest China, where well-developed caves and highly-connected underground flow channels are abundant. One of the important characteristics of karst formations in semi-arid areas is that caves and subsurface rivers do not exist (Yuan, 1994). As a consequence, the ratios of the maximum to minimum spring flow in semi-arid areas are generally from 1.24 to 5.89, whereas the ratios are from 10 to 1000 in humid regions (He et al., 1997). These ratios are indicative of a rapid response of the karst aquifer in the humid region. The areal extent of a karst ground water system in semi-arid regions is usually up to several thousands of square kilometers, and the pores in the carbonate rocks provide extensive storage for precipitation. The spring discharges in these semi-arid regions generally lag behind precipitation by 2–10 years (Han et al., 1993; Ma et al., 2004).

Different models have been developed to study the influence of climate change on groundwater. Studies have shown that the impacts of climate change on groundwater are site-specific. For example, an integrated hydrological model (MOHISE) has been developed to study the impact of climate change on the hydrological cycle in several representative water basins in Belgium (Brouyère et al., 2004). Their study shows no enhancement of seasonal changes to ground water levels in this region. But a cross-correlation analysis by Chen et al. (2004), which examines the importance of climate on groundwater level variation, indicates that both precipitation and annual mean temperature display a strong correlation with annual groundwater levels in the upper carbonate aquifer in southern Manitoba, Canada. Chen et al. (2002) developed an empirical model that linked climate variables to groundwater level predicted variations in groundwater level in response to different climate scenarios in a climate change impact assessment. Furthermore, 2·CO₂ climate scenarios were developed by Loáiciga et al. (2000) to demonstrate the effects of climate variability on surface hydrology and aquifer dynamics. These climate scenarios

used alternative numerical simulation models that were calibrated using data from the Edwards Balcones Fault Zone (BFZ) aquifer, TX, USA. Their simulations indicated that 2·CO₂ climatic conditions could exacerbate negative impacts and water shortages in the Edwards BFZ aquifer, even if pumping does not increase above its present average rate.

A detailed investigation of potential impacts of climate change on groundwater necessitates detailed characterization of the vadose zone and aquifers. Detailed characterizations of spatially-varying hydraulic properties of field sites remain one of the main challenges for successful prediction of flow and transport in the vadose zone (Hughson and Yeh, 1998). As a result, simplified models are often adopted for this purpose. Based upon the ideas of gray system theory developed by Deng (1982), the groundwater system is more accurately viewed as a gray system where part of the information is clearly understood and part is obscure. The purpose of the current project is to study the response of karst aquifers to climate change in semi-arid areas by employing gray system models. As a representative of karst springs in semi-arid areas of China, the Liulin Springs were selected as our study area.

The hydrogeological setting of the Liulin Springs

The Liulin Springs site is located in the Sanchuan River Valley, in the Liulin groundwater basin, east Liulin County of Shanxi Province (Fig. 1). The Sanchuan River is a branch of the Yellow River. The springs consist of about 100 spring points, which can be classified into five groups, namely, Longmenhui Springs, Shangqinglong Springs, Zhaidongquan Springs, Yangjiaxiang Springs, and Liujiageda Springs. The springs are distributed along the Sanchuan River bank where the elevation ranges from 790 to 801 m, and the zone of spring discharge stretches ≈2 km along the Sanchuan River.

The Liulin groundwater basin covers an area of 5100 km². The Liulin Springs flow at an annual average rate of 3.38 m³/s (an average over 1957–2002). In the Liulin Basin the annual mean air temperature is 9.2 °C and the annual average precipitation is 506 mm with 60–70% of this total precipitation occurring during the peak rainfall months (i.e., July, August and September). Major rivers in the Liulin groundwater basin include the Beichuan, Dongchuan, and Nanchuan Rivers. All of these rivers converge to form the Sanchuan River at Jiaokou Town (see Fig. 1).

The ground surface elevation in the Liulin groundwater basin ranges from 660 to 1200 m. The eastern part of the basin is higher than the western part; general topography of the basin inclines to the west. The main aquifers of the basin are comprised of karstic Cambrian and Ordovician limestone, and porous Quaternary sandstone sediment (Fig. 2). On a regional scale, the limestone and Quaternary sediment aquifers are hydraulically connected, although Permian carboniferous sandstone and shale exist between them.

Precipitation is believed to be the primary source of recharge to the aquifer. Groundwater recharge occurs mainly from infiltration of precipitation into outcropping limestone

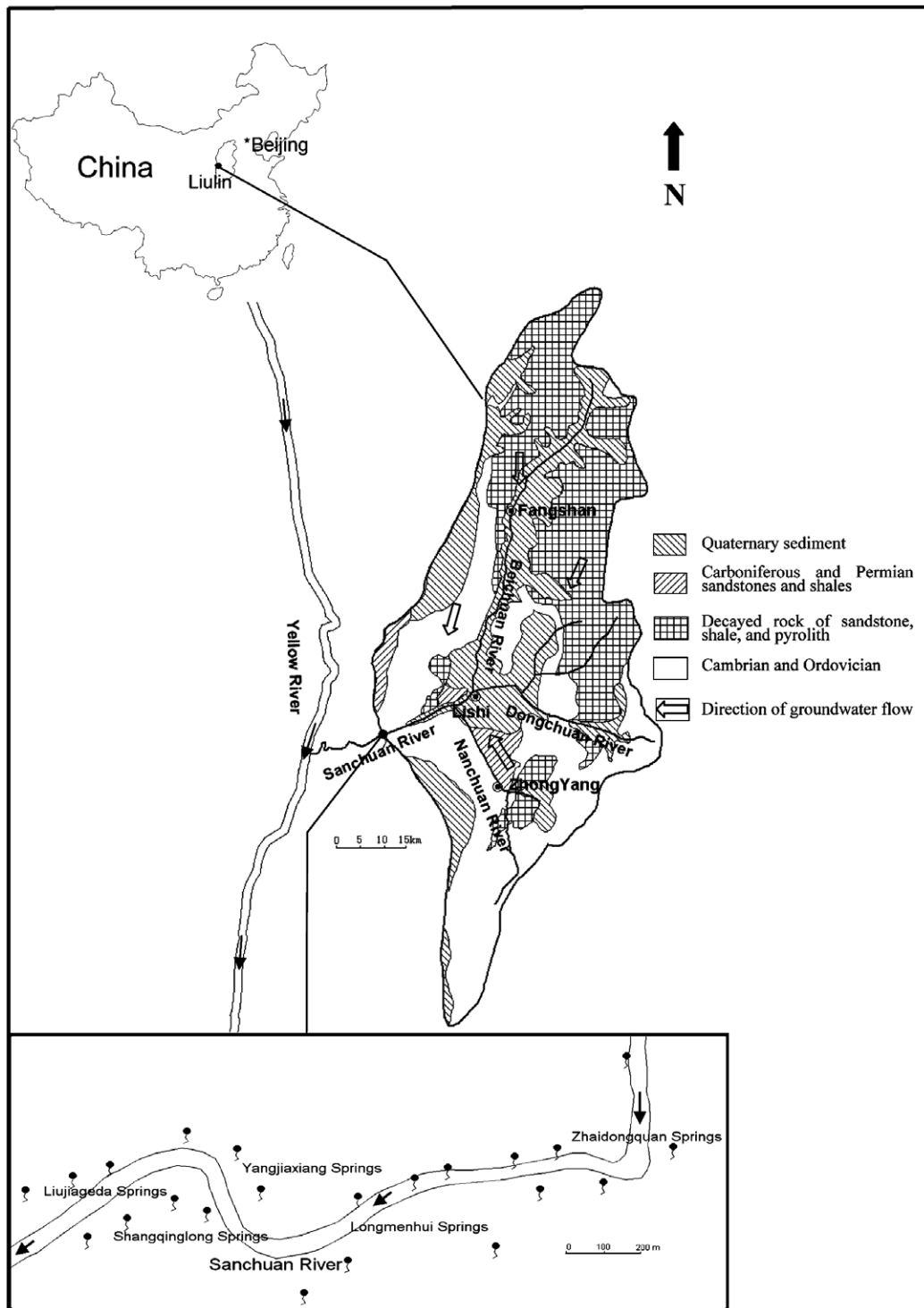


Figure 1 Location of the Liulin Springs, and a simplified hydrogeology map of the Liulin Basin, with an enlarged view of the distribution of springs.

areas, river beds, and sinkholes. Controlled by the regional geologic structure, groundwater generally flows toward the west. In the west, groundwater flow encounters Carboniferous and Permian sandstone and shale of lower permeability, and discharges as springs to the surface in the Sanchuan River Valley.

Since the early 1970s, groundwater in the Liulin Basin has been exploited for irrigation, municipal use, and industrial water supplies. Meanwhile, the groundwater level has declined, and spring flow has decreased (Fig. 3). Such decreases in groundwater level and spring flow have become a major concern of residents of the basin.

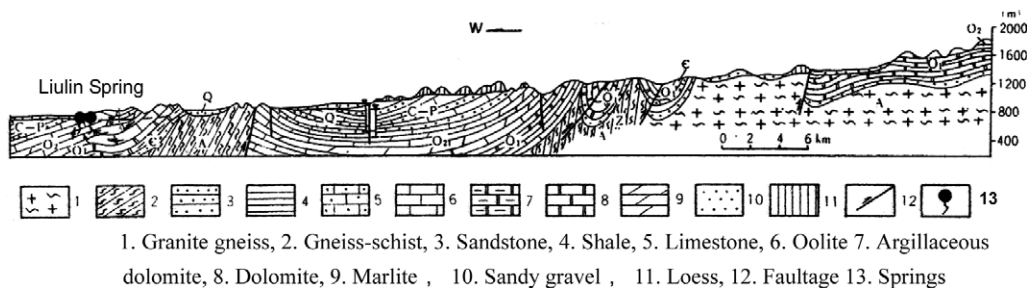


Figure 2 A cross-sectional geologic map of the Liulin Spring Basin.

Gray system theory

In gray system theory (Deng, 1982, 1985), color is used as an index to represent the level of the recognition of a system. White is used to denote a completely known system, and black represents a completely unknown system. The gray system theory generally includes: (a) gray incidence analysis, which compares and evaluates a system’s factor behaviors; (b) system modeling, which predicts the behavior of the system. Hydrologic and geologic systems are, by their nature, complex and usually heterogeneous, and it is very rare for such systems to be completely understood in all their complexity. Accordingly, the karst system that we are investigating is ideal for the application of gray system theory (Lee and Wang, 1998; Xia, 2000).

In the karst region, precipitation and runoff often enter the subsurface through infiltration into the vadose zone or sink holes, fissures, etc. and become ground water. While in the subsurface, water may flow through the porous matrix, fissures, fractures, or stream channels under either saturated or unsaturated conditions. The flow may be turbulent or laminar. Ground water in karst aquifers may emerge as springs, become surface water, and then reenter the subsurface. These processes are complex and are difficult to describe with accuracy and precision using mathematical models. In addition, these processes are controlled by meteorological, hydrological, topographical, physiographical, and geological factors, as well as by vegetation and human activities. These factors often exhibit significant temporal and spatial variability and cannot be accurately quantified. Finally, there is very sparse historical data on variables such as: recharge from precipitation; outflow (e.g., spring flow and regional flow); and aquifer responses such as ground water level variation, all of which are input variables to the aquifer. As a consequence, a gray system approach is appropriate for the investigation.

Description of gray incidence analysis (GIA)

Gray incidence analysis (GIA) is a method to estimate the degree of interaction among factors that govern system development. If the trend of two factors is consistent, the degree of gray incidence will be large. If the trend of the two factors is less well-defined, the degree of gray incidence will be small (Liu et al., 2004; Wang et al., 1989, 2001b). While GIA is similar to the well-known cross-correlation analysis, it does not rely on any statistical concept.

A gray relation space is a binary set denoted by (X, Γ) , where X is a collection composed of sequences including n entries, X_0 is called a mapping quantity of the system behavior (e.g., spring flow record), X_i are behavioral sequences of relevant factors to be compared (e.g., precipitation, evapotranspiration, groundwater level records, etc.), and Γ is a gray incidence map set (e.g., the relation between the spring flow and precipitation records). The system characteristic behavior sequence X_0 and relevant behavioral sequences X_τ can be presented in a form of series as:

$$X_0 = (x_0(t)), \quad t = 1, 2, \dots, n \tag{1}$$

$$X_\tau = (x_\tau(t - \tau)), \quad \tau = 1, 2, \dots, m \tag{2}$$

where t is time and τ is the time-lag between X_0 and X_τ . During the GIA analysis, we use the normalized value of X_0 and X_τ

$$X'_0 = (x'_0(t)) = \frac{X_0}{\bar{x}_0}, \quad t = 1, 2, \dots, n \tag{3}$$

where $\bar{x}_0 = \frac{1}{n} \sum_{t=1}^n x_0(t)$;

$$X'_\tau = (x'_\tau(t - \tau)) = \frac{X_\tau}{\bar{x}_\tau}, \quad t = 1, 2, \dots, n, \quad \tau = 1, 2, \dots, m \tag{4}$$

where $\bar{x}_\tau = \frac{1}{n} \sum_{t=1}^n x_\tau(t - \tau)$

The expression of the incidence coefficient of X_τ with respect to X_0 at the point t is defined as:

$$\gamma_{0\tau}(t) = \frac{\gamma(x'_0(t), x'_\tau(t - \tau))}{\frac{\min_{\tau} \min_t |x'_0(t) - x'_\tau(t - \tau)| - \max_{\tau} \max_t |x'_0(t) - x'_\tau(t - \tau)|}{|x'_0(t) - x'_\tau(t - \tau)| - \max_{\tau} \max_t |x'_0(t) - x'_\tau(t - \tau)|}} \tag{5}$$

where the degree of gray incidence of X_τ with respect to X_0 , is defined as

$$\gamma(X_0, X_\tau) = \frac{1}{n} \sum_{t=1}^n \gamma(x'_0(t), x'_\tau(t - \tau)) \tag{6}$$

After the degrees of gray incidence are determined, the gray incidence order is determined through comparing $\gamma(X_0, X_\tau)$. The gray incidence order reflects the relative intimate grade between X_0 and X_τ .

The GM(1,2) simulation model

After the gray incidence analysis, the gray system model, GM(1,2), is used to forecast the system characteristic behavior. The GM(1,2) model is composed of first order, two variables, and differential time series equations. To

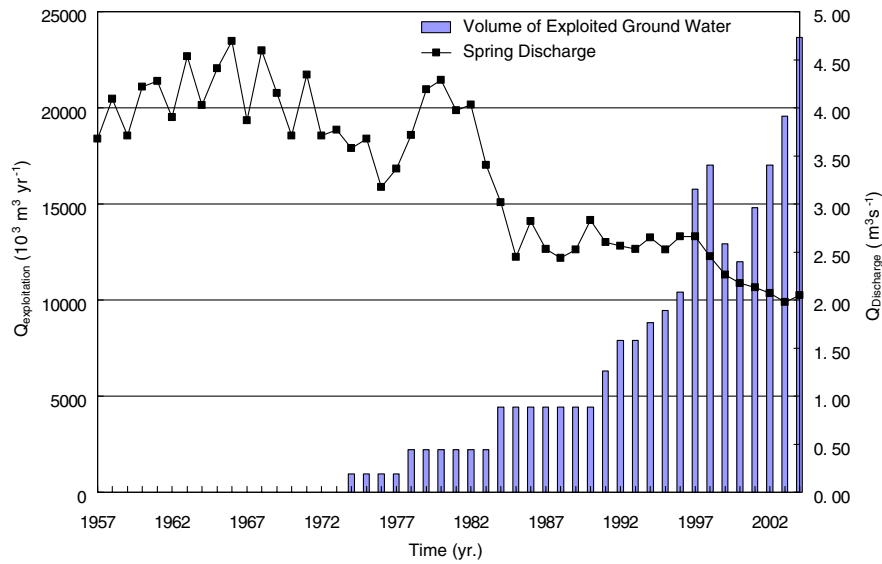


Figure 3 Time series of the estimated spring discharge and volume of exploited groundwater during 1957–2004.

set up the grey model, a set of disordered raw data is first transformed into a new set by means of the accumulated generating operation (AGO). A set of differential equations is then set up to serve as the governing equations for the transformed information. Coefficients of the governing equations are then determined. Subsequently, the equations and the coefficients are used to forecast the system behavior (Liu and Lin, 1998; Deng et al., 1988). The detail of the model is given below.

Suppose that

$$X_1^{(0)} = (x_1^{(0)}(t)), \quad t = 1, 2, \dots, n \tag{7}$$

is a data sequence of a system’s characteristics (e.g., spring flow), and

$$X_2^{(0)} = (x_2^{(0)}(t - \tau)), \quad t = 1, 2, \dots, n \tag{8}$$

is a sequence of relevant factors (e.g., precipitation). And there is time-lag τ existence between $X_1^{(0)}$ and $X_2^{(0)}$. Now, we define that

$$X_1^{(1)} = (x_1^{(1)}(t)), \quad t = 1, 2, \dots, n \tag{9}$$

and

$$X_2^{(1)} = (x_2^{(1)}(t - \tau)), \quad t = 1, 2, \dots, n \tag{10}$$

are the sequences obtained through the AGO, in which

$$x_1^{(1)}(t) = \sum_{i=1}^t x_1^{(0)}(i), \quad t = 1, 2, \dots, n \tag{11}$$

$$x_2^{(1)}(t - \tau) = \sum_{i=1}^t x_2^{(0)}(i - \tau), \quad t = 1, 2, \dots, n \tag{12}$$

Next, the grey derivative of $X_1^{(1)}$ is then defined as

$$\frac{dx_1^{(1)}(t)}{dt} = x_1^{(1)}(t) - x_1^{(1)}(t - 1) = x_1^{(0)}(t) \tag{13}$$

and we define a GM(1,2) gray differential equation:

$$\frac{dx_1^{(1)}(t)}{dt} + ax_1^{(1)}(t) = x_1^{(0)}(t) + ax_1^{(1)}(t) = bx_2^{(1)}(t - \tau) \tag{14}$$

In a GM(1,2) gray differential equation, $-a$ is called the development coefficient of the system, $bx_2^{(1)}(t - \tau)$ the driving term, and b the driving coefficient. Then the sequence of parameters is given as

$$\hat{a} = [a, b]^T \tag{15}$$

To estimate the sequence of parameters, a least squares regression is used. That is, consider that

$$B = \begin{bmatrix} 0.5x_1^{(1)}(2) + 0.5x_1^{(1)}(1), & x_2^{(1)}(2) \\ 0.5x_1^{(1)}(3) + 0.5x_1^{(1)}(2), & x_2^{(1)}(3) \\ \vdots & \vdots \\ 0.5x_1^{(1)}(n) + 0.5x_1^{(1)}(n - 1), & x_2^{(1)}(n) \end{bmatrix} \tag{16}$$

$$Y = \begin{bmatrix} x_1^{(0)}(2) \\ x_1^{(0)}(3) \\ \vdots \\ x_1^{(0)}(n) \end{bmatrix} \tag{17}$$

Then the least squares estimation for the sequence of parameters is

$$\hat{a} = (a, b)^T = (B^T B)^{-1} B^T Y \tag{18}$$

Once the sequence of parameters is known, the solution to Eq. (14) is given as

$$\hat{x}_1^{(1)}(t) = \left(x_1^{(0)}(1) - \frac{b}{a} x_2^{(1)}(t - \tau) \right) e^{-at} + \frac{b}{a} x_2^{(1)}(t - \tau) \tag{19}$$

Then the approximate time response sequence of the GM(1,2) gray differential Eq. (14), an approximation of Eq. (19), may be written as:

$$\hat{x}_1^{(1)}(t + 1) = \left(x_1^{(0)}(1) - \frac{b}{a} x_2^{(1)}(t - \tau + 1) \right) e^{-at} + \frac{b}{a} x_2^{(1)}(t - \tau + 1) \tag{20}$$

where $\hat{x}_1^{(1)}(t + 1)$ represents our estimate of $x_1^{(1)}(t + 1)$, and $x_1^{(1)}(0)$ is taken to be $x_1^{(0)}(1)$. Finally, we obtain our

prediction of spring flow at $t + 1$ using the inverse accumulated generating operation (IAGO):

$$\hat{x}_1^{(0)}(t + 1) = \hat{x}_1^{(1)}(t + 1) - \hat{x}_1^{(1)}(t) \tag{21}$$

Application

The time-lag between spring discharge and precipitation

According to GIA discussed above, the discharge of the Liulin Springs can be thought of as the mapping quantity of the system behavior, X_0 , which was constructed using the annual average discharges of Liulin Springs from 1970 to 2002. The spring discharge refers to the estimated discharge, which is calculated by adding the observed spring discharge to the volume of exploited karst water from wells. The relevant factor behavioral sequences, X_i , were constructed using the annual precipitation. Different 1-year time lags, up to 7 years, were assigned to X_i and therefore, seven sequences were constructed. The degrees of gray incidence between discharge and precipitation of different time-lags at the Liulin Springs Basin were determined. The results are presented in Table 1.

From Table 1, the gray incidence order in different time-lags are obtained: $\gamma_4 > \gamma_3 > \gamma_6 > \gamma_7 > \gamma_1 > \gamma_2 > \gamma_5$. The largest degree of gray incidence was 0.827, when the time-lag was 4 years. Therefore the time-lag between spring discharge and precipitation at the Liulin Springs is about 4 years. It means that the karst groundwater residence time in the Liulin Springs Basin is about 4 years. In other words, the Liulin Springs might be slow-response springs according to the classification of karst springs by White (1988). This response time may appear too long according to one’s general knowledge of karst hydrology. But one must consider the fact that the study area is located in a semi-arid region in the Loess Plateau China, and most carbonate aquifers are overlain by thick Carboniferous–Permian sandstone and shale, and Quaternary sediments in the Liulin Springs Basin (Wang et al., 2001a). Accordingly, the groundwater will travel a long distance from points of infiltration to a spring outlet, and will thus have a long residence time (Loáiciga, 2004). As an example other than the Liulin Springs, the residence time in the Shentou Springs Basin, Shanxi Province, China, is about 10 years (Ma et al., 2004). Like the Liulin Springs Basin, the Shentou Springs Basin is also in a semi-arid climate, and the karst aquifer is also overlain by thick sediments.

The GM(1,2) time-lag simulation model of the Liulin Springs discharge

To analyze the Liulin Springs system, we chose the annual average spring discharge to be the system’s characteristic

sequence $X_1^{(0)}$, and let the annual precipitation be the relative factor sequence $\tilde{X}_2^{(0)}$. Application of the GM(1,2) model requires one to choose the most suitable sequences of $X_1^{(0)}$ and $\tilde{X}_2^{(0)}$. Generally, gray system theory does not consider longer sequences of $X_1^{(0)}$ and $\tilde{X}_2^{(0)}$ to be better. Choosing the time series that: (a) is consistent with the future developing tendency; (b) that can reflect the future variation of spring flow, may be the most critical requirements. The gray system theory assumes that the information in a specific period can be divided into three phases: the past, present and future. The past information influences the present information directly, which, in turn, impacts the future tendency. That is to say, the present information can be treated as an intermediary that connects with the past with the future information, and the actions of the past information on influencing future tendencies is achieved by the analysis of present information, so the effect of the past information on the future is indirect.

We used the spring discharges and precipitation data from 1957 to 2002 to set up the GM(1,2) simulation model. Then, the data of 2003 and 2004 were used as test of the model prediction. Fig. 4 is a plot of spring discharges and precipitation data from 1957 to 2002. This figure indicates that the spring flow has declined since 1985. Specifically, the annual mean spring discharge from 1957 to 1984 was 3.93 m³/s, and from 1985 to 2002 was reduced to 2.49 m³/s. Considering a time-lag of 4 years between spring discharge and precipitation, the precipitation from 1957 to 1980 was found to be 518.6 mm and from 1981 to 2002 was reduced to 473.8 mm. The discharge record of the Liulin Springs reflected the effect of decreased precipitation, although the effect on discharge was delayed by a few years. Accordingly, the discharge data were divided into two phases at the year 1985. In the first phase (1957–1984) the discharges were generally stable at a high level over time and in the second phase (1985–2002) the spring flow declined.

According to gray system theory, the first phase of the Liulin Springs discharge data (1957–1984) can be thought of as the past information, while the secondary phase data (1985–2002) are defined as the present information. Our assumption is that the secondary phase of the Liulin Springs discharge can at least adequately predict the springs’ discharge. Spring discharge data from 1985 to 2002 were selected as the optimal $X_1^{(0)}$ sequences to set up the GM(1,2) model. Accordingly, the $\tilde{X}_2^{(0)}$ sequence was taken from the precipitation data from 1981 to 1998, considering the time-lag between $X_1^{(0)}$ and $\tilde{X}_2^{(0)}$ was about 4 years.

Precipitation becomes ground water when it sinks into the ground, and karst ground water becomes surface water when it emerges from springs. In other words, precipitation is the input to the system, and spring discharge is the output of the system. Precipitation data were collected from a sparse gauging station network in the basin, and may involve a large degree of uncertainty. As a result, the annual precip-

Table 1 The degrees of gray incidence between discharges and precipitation for different time-lags

Time-lag (year)	1	2	3	4	5	6	7
Degrees γ_i	0.821	0.821	0.823	0.827	0.818	0.822	0.822

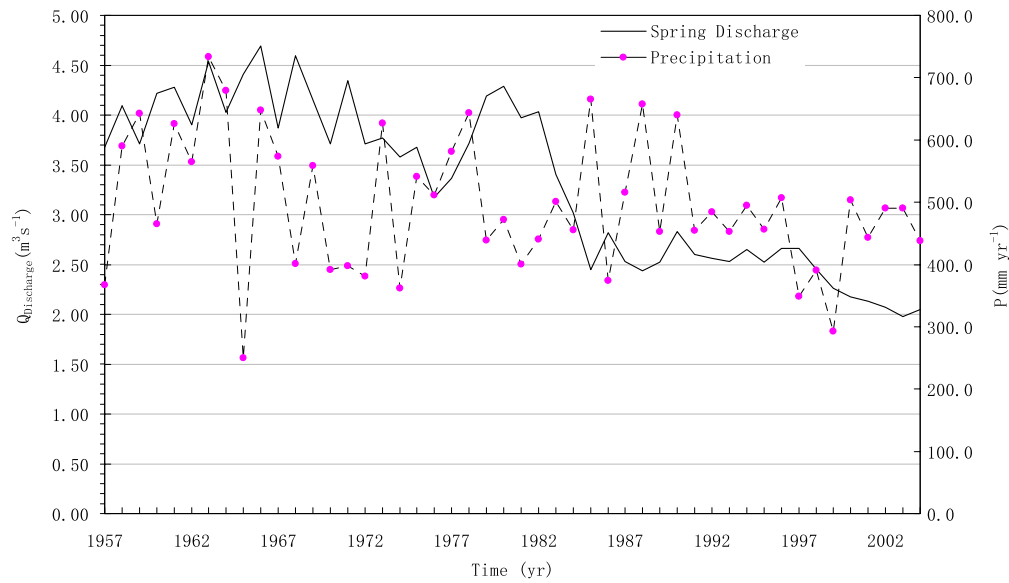


Figure 4 Time series of the estimated spring flow and recorded precipitation.

itation series $\tilde{X}_2^{(0)}$ is preprocessed by a moving average method

$$x_2^{(0)}(1) = \frac{3\tilde{x}_2^{(0)}(1) + \tilde{x}_2^{(0)}(2)}{4} \quad (22)$$

$$x_2^{(0)}(t) = \frac{\tilde{x}_2^{(0)}(t-1) + 2\tilde{x}_2^{(0)}(t) + \tilde{x}_2^{(0)}(t+1)}{4}, \quad t = 2, \dots, n-1 \quad (23)$$

$$x_2^{(0)}(n) = \frac{\tilde{x}_2^{(0)}(n-1) + 3\tilde{x}_2^{(0)}(n)}{4} \quad (24)$$

Then we have $X_2^{(0)} = (x_2^{(0)}(t))$, $t = 1, 2, \dots, n$.

According to the sequences of $X_1^{(0)}$ and $X_2^{(0)}$, we set up the GM(1,2) time-lag model for the Liulin Springs. The param-

eters of the GM(1,2) time-lag model are $\hat{a} = (a, b)^T = \begin{pmatrix} 3.3165 \\ 0.0169 \end{pmatrix}$ with the time-lag $\tau = 4$. The estimation model is thus expressed as

$$\frac{dx_1^{(1)}(t)}{dt} + 3.3165x_1^{(1)}(t) = 0.0169x_2^{(1)}(t - \tau) \quad (25)$$

and the approximate time-sequence response is

$$\hat{x}_1^{(1)}(t+1) = (2.45 - 0.0051x_2^{(1)}(t - \tau + 1))e^{-3.3165t} + 0.0051x_2^{(1)}(t - \tau + 1) \quad (26)$$

From the inverse accumulated generating operation (IAGO) restoration,

Table 2 Testing of the GM(1,2) simulation model of the discharge at the Liulin Springs

Year	$x_1^{(0)}(t)$ Raw discharge (m^3/s)	$\hat{x}_1^{(0)}(t)$ Simulated discharge (m^3/s)	$\varepsilon(t) = x_1^{(0)}(t) - \hat{x}_1^{(0)}(t)$ Residual error	$\Delta(t) = \frac{ \varepsilon(t) }{x_1^{(0)}(t)}$ Relative error (%)
1985	2.45	2.45	0	0
1986	2.82	2.28	0.54	19.13
1987	2.53	2.51	0.02	0.83
1988	2.44	2.49	-0.05	2.23
1989	2.52	2.56	-0.04	1.43
1990	2.83	2.72	0.11	3.90
1991	2.60	2.72	-0.12	4.60
1992	2.56	2.63	-0.07	2.72
1993	2.53	2.77	-0.24	9.43
1994	2.65	2.74	-0.09	3.38
1995	2.52	2.53	-0.01	0.31
1996	2.66	2.58	0.08	3.03
1997	2.66	2.39	0.27	10.19
1998	2.45	2.44	0.01	0.58
1999	2.26	2.3	-0.04	1.77
2000	2.18	2.24	-0.06	2.93
2001	2.13	2.09	0.04	1.88
2002	2.07	2.07	0	0

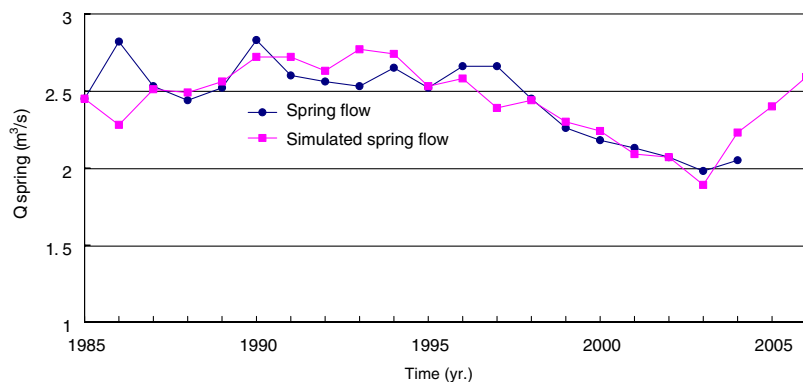


Figure 5 Simulation and estimated spring discharge.

$$\hat{x}_1^{(0)}(t+1) = \hat{x}_1^{(1)}(t+1) - \hat{x}_1^{(1)}(t) \quad (27)$$

the simulated discharges of the Liulin Springs were calculated (Table 2). Results of simulated spring discharge using the GM(1,2) model are presented in Fig. 5 and Table 2.

Model testing

In Table 2, the minimum average relative error is 0% in 2002 and the maximum average relative error is 19.13% in 1986. The average relative error is 3.80% and the simulation curve is in substantial agreement with the spring discharge data (Fig. 5). Therefore, the GM(1,2) time-lag simulation model of spring discharge can adequately simulate and forecast spring discharge.

Prediction of discharge at the Liulin Springs

Using annual average precipitation from 1999 to 2002 as input to Eq. (26), the predicted spring discharges from Eq. (27) for the years 2003–2006 are found to be 1.89, 2.23, 2.40 and 2.59 m³/s, respectively. The accuracy of these predictions is supported by the estimated spring flow data: 1.98 and 2.05 m³/s for 2003 and 2004, respectively. The errors are 4.5% and 8.8%, for 2003 and 2004, respectively.

Discussion

Gray system theory was applied to study the responses of karst spring flow to climate change in a semi-arid region in China. The Liulin Springs were selected as a representative case. Through GIA, we determined that the time-lag between spring discharge and precipitation was 4 years at the Liulin Springs. Our data are in agreement with White's (1988) karst spring classification, which would identify the Liulin Springs as a system of slow-response springs. A slow response to precipitation is also considered one of characteristics of karst springs in semi-arid regions in China (Yuan, 1994).

The reduction of precipitation in the secondary phase strongly affected the decline of spring discharge. Therefore, the influence of climate change on spring discharge at the Liulin Springs is long-term and evident. Based on the GM(1,2) time-lag simulation model, we conclude that spring discharge is positively correlated with precipitation. It thus seems realistic to reemphasize that climate change

is likely to have a significant impact on karst aquifers at the Liulin Springs.

In the Liulin Springs Basin, karst water exploitation began in the early 1970s and was one of the reasons that spring discharge declined. Groundwater exploitation and coal mining in the Liulin Springs Basin disturbed the groundwater flow, which impacted the recharge of precipitation to the karst aquifer, and in turn, could indirectly cause the decrease of spring discharge. Fig. 3 shows that since the 1970s, ground water exploitation has exhibited an increasing trend over the last 30 years, corresponding to economic and population growth. It would seem that there is a clear connection between the reduction of spring discharge and groundwater exploitation. However, the difference between the first and second phases of average discharge at the Liulin Springs is 1.44 m³/s, and the average exploitation was only 0.24 m³/s. For these reasons, as compared with the effect of possible climate change scenarios, and while still critical to the evaluation of water resources and water resources planning, human activities are secondary in affecting spring discharge at the Liulin Springs.

Conclusions

By applying gray incidence theory to the Liulin Springs, we have learned that the time-lag between spring discharge and precipitation is about 4 years. Using the GM(1,2) model, we set up the GM(1,2) time-lag simulation model of spring discharges using precipitation as input, and predicted that the discharges of the Liulin Springs are 1.89, 2.23, 2.40 and 2.59 m³/s in the years 2003–2006, respectively. The model predictions are in agreement with observed data. We have shown that gray system theory not only provides an understanding of the uncertainties of a karst water system, but also can be used to analyze and simulate a karst water system.

In the study, influences of climate change on karst aquifers in the semi-arid regions were presented and discussed. This study indicates that the karst aquifers at the Liulin Springs respond remarkably to climate changes, in particular to changes in precipitation input. Meteorological observations in semi-arid areas of China indicated that the decrease in precipitation in the 1970s and 1980s was regional (Ma et al., 2004). On this basis, it could be inferred that a continuous decline of karst ground water level and the

waning of spring discharges in semi-arid regions of China might be largely a response of the groundwater system to the decrease in regional precipitation. Human activities are secondary in affecting spring discharge at the Liulin Springs when compared with climatic effects.

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