The Intrinsic Hypothesis

It is a less stringent hypothesis than the second-order stationary assumption. It assumes that even if the variance of Z is not finite, the variance of the first-order increments of Z is finite and the increments are second-order stationary, i.e., that [Z(x + h) - Z(x)] satisfies

1) E [Z(x + h) - Z(x)] = m(h)
2) var [Z(x + h) - Z(x)] = 2γ(h)

In other words, the difference between Zs at different locations will have a constant mean and variance as long as the Zs are separated by the same distance h. The mean and the variance of the difference are functions of h, separation distance, not x, and

$$\gamma(h) = \frac{1}{2} VAR[Z(x+h) - Z(x)] \qquad \text{variogram (semi-variogram)}$$

The properties of the variogram are: 1) the value of the variogram at the origin is zero, $\gamma(0)=0$; 2) the values of the variogram are positive, $\gamma(h)\geq 0$, and 3) the variogram is an even function, $\gamma(h)=\gamma(-h)$.

Now, if we assume

$$E[Z(x+h) - Z(x)] = m(h) = 0$$

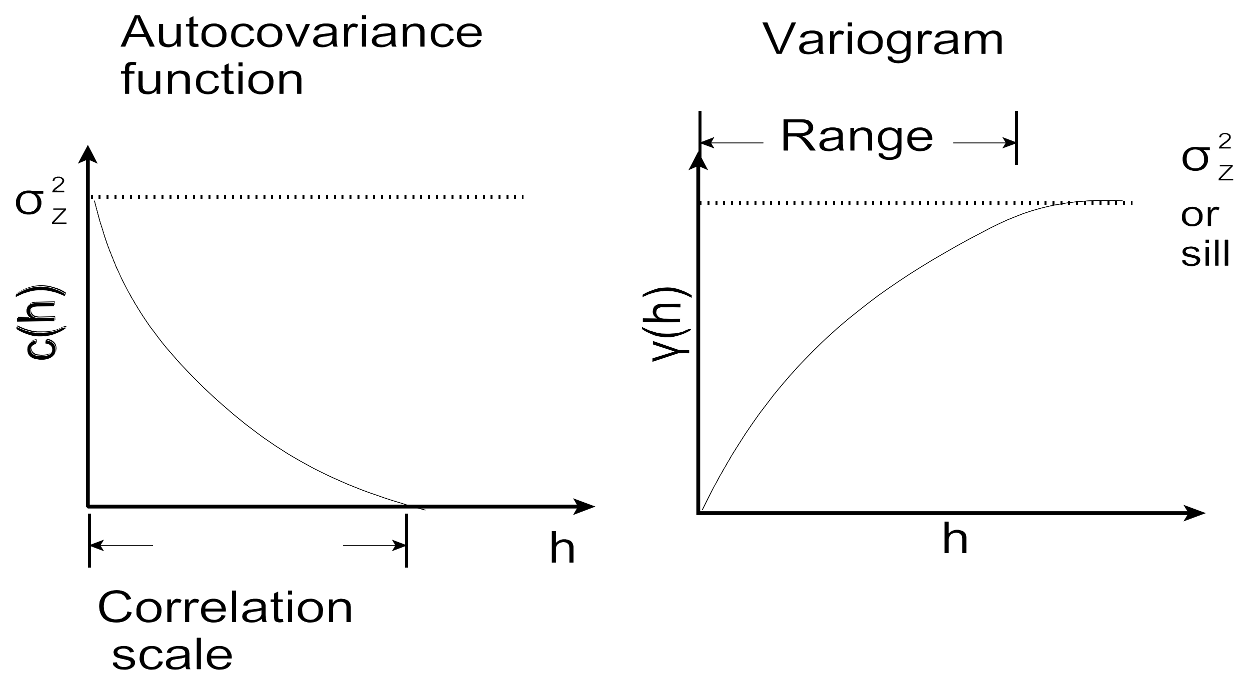
then

$$\begin{aligned}\gamma(h) &= \frac{1}{2} E\left\{\left[Z(x+h) - Z(h)\right]^2\right\} \\ &= \frac{1}{2}\left\{E\left[Z(x+h)^2\right] - 2E[Z(x+h)Z(h)] + E\left[Z(h)^2\right]\right\}\end{aligned}$$

If Z is a second-order stationary process, the variogram can be related to the covariance.

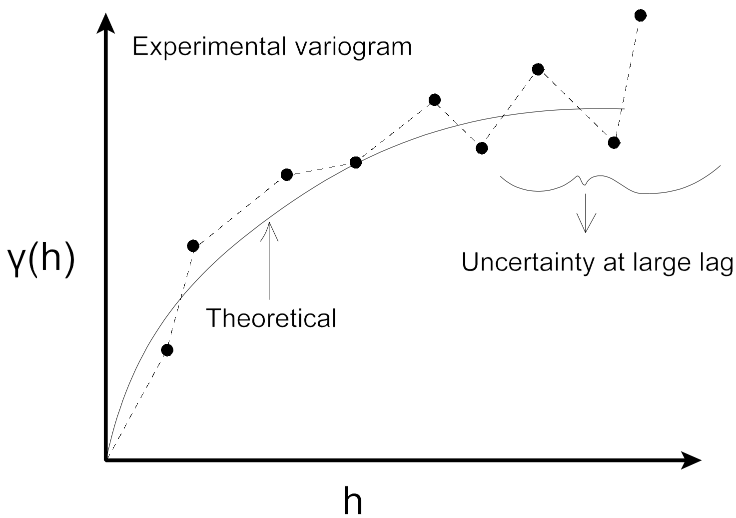
$$\gamma(h) = C(0) - C(h)$$

The variogram, in this case, is a mirror image of the autocovariance function as illustrated in the figure below.


Autocovariance function
σ 2 z
c(h)
h
Correlation scale
Variogram
Range
σ 2 z
or
sill
γ(h)
h

§ Variogram

In general, variograms of hydraulic properties are estimated from observed data using the above formula or using software packages such Geo-EAS.

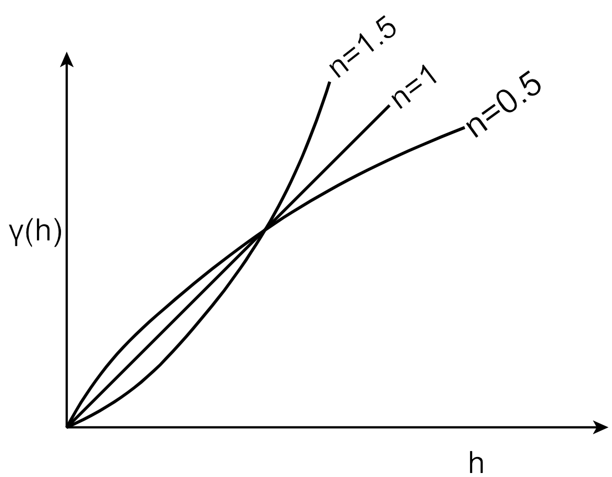


§ Theoretical Variogram

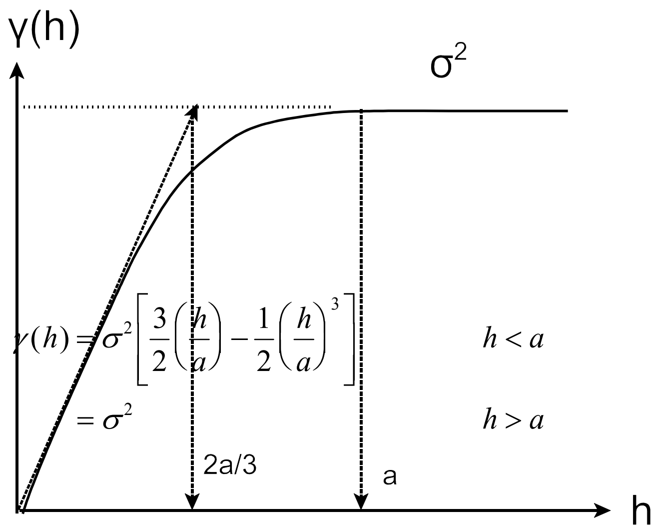
(positive definite function)

1. linear variogram model

$$\gamma(h) = \sigma^2 |h|^n$$

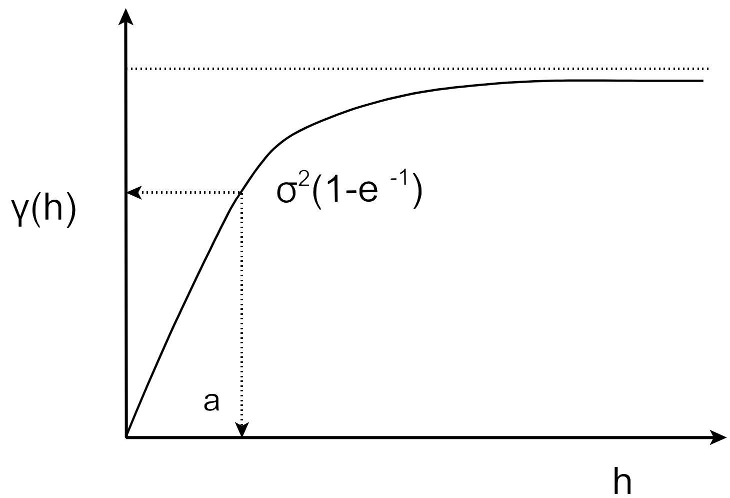


2. Spherical variogram model:



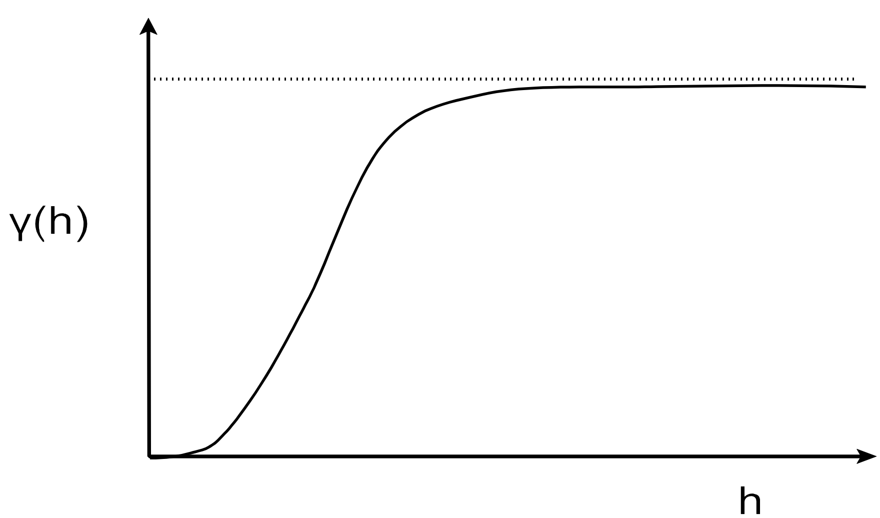
3. Exponential Model

$$\sigma^2 \left[1 - \exp\left(\frac{-|h|}{a} \right) \right]$$



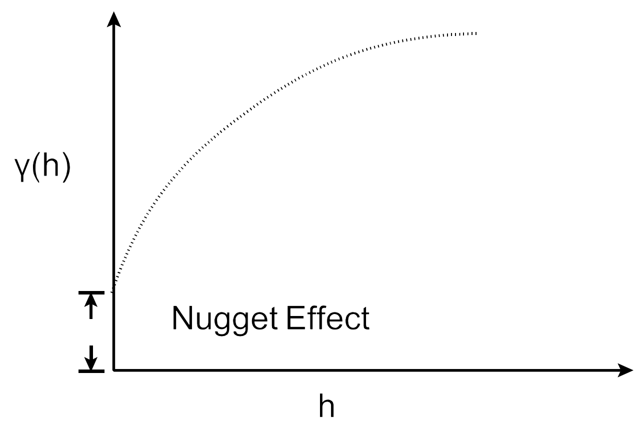
4. Gaussian Model

$$\sigma^2\left(1 - \exp\left[-\left(\frac{h}{a}\right)^2\right]\right)$$



There are many other models that will satisfy the positive definite requirement (de Marsily, 1986 and Isaaks and Srivastava, 1989).

Nugget Effect: The variogram is discontinuous near the origin.



1) measurement errors

2) Sampling interval is too large

$$\gamma(h) = c[1 - \delta(h)] + \gamma'(h)$$

where

$$\delta(h) = 1 \quad \text{if } h = 0 \text{ and } = 0 \text{ if } h \neq 0$$

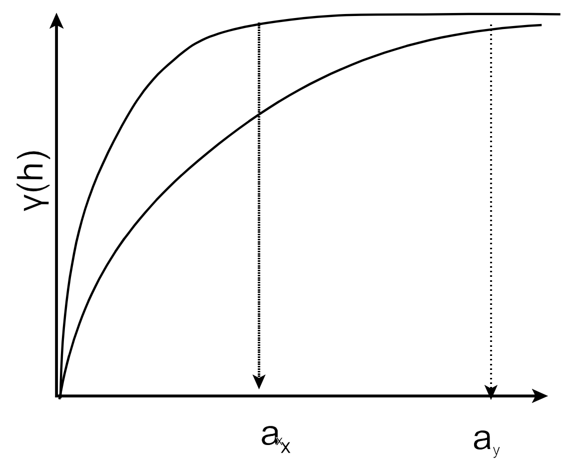
Anisotropic Variogram:

Similar to correlation scales, the variogram may have different ranges in different directions due to geological deposition processes. Again, it has been observed that most hydraulic conductivity data tend to show a longer range in the horizontal plane than in the vertical direction. This type of the variogram is called anisotropic variogram. Theoretically, the variograms in different directions should have the same sill but different ranges. For example,

Head → longer range in the direction perpendicular to mean flow direction

→ small in the direction of mean flow.

Log Transformed hydraulic conductivity: The range in the horizontal direction is much larger than the range in the vertical direction–implications of layering structure of geological formation.



If the variograms in different directions have the same sill values but different ranges. This anisotropy is called the geometric anisotropy. Some field data, however, show that the variograms may have different sill values and the ranges values in different directions. The anisotropy is called the zonal anisotropy. This type anisotropy is mainly attributed to insufficient samples. That is, the sample area is small compared to the anisotropic structure of the geological formation. For example, if one intensively samples only over a 50 m x 50 m x 50m volume of a geological formation which has layers that extend horizontally over 500 m, the analysis of the samples likely yields a zonal anisotropy.

In general, T and K are assumed to have lognormal distributions. That is, lnT and lnK are normally distributed. As a result, the linear predictor (e.g., kriging) yields exact condtional expectation (mean).

Questions.

There are many theoretical variogram models. One logical question to ask is: does one have to carefully select the exact model for the experimental variogrm?