DARCY'S LAW FOR FIELD-SCALE PROBLEMS (535-L5.wpd)

Scales and Heterogeneity

Hydrologic properties of geologic media often exhibit a high degree of spatial variability at various scales due to the heterogeneous nature of geological formations. For laboratory scale problems (i.e., small cores, soil columns, and sand boxes), variations in pore size, pore geometry, and tortuosity of pore channels are the major sources of heterogeneity. They are called **laboratory-scale heterogeneity**. Microstratification, foliation, cracks, and roots are also some possible heterogeneities at this scale. As our observation scale increases to a field, stratification or layering in a geologic formation becomes the dominant heterogeneity, which is often classified as **field-scale heterogeneity**. At an even larger observation scale, the regional-scale heterogeneity represents the variations of geologic formations or facies. Variations among sedimentary basins are then categorized as **global-scale heterogeneity**.

Fundamental theories for flow and solute transport through porous media are essentially derived for laboratory-scale heterogeneity. When we attempt to apply these theories to the vadose zone and aquifers, comprising heterogeneities of many different scales, we encounter the scale problem. For analyzing flow and solute transport of field-scale problems, two methodologies based on the theories for the laboratory-scale problem have evolved, namely, classical analysis and stochastic analysis, to be discussed later in the lecture.

Classical Conceptual Models

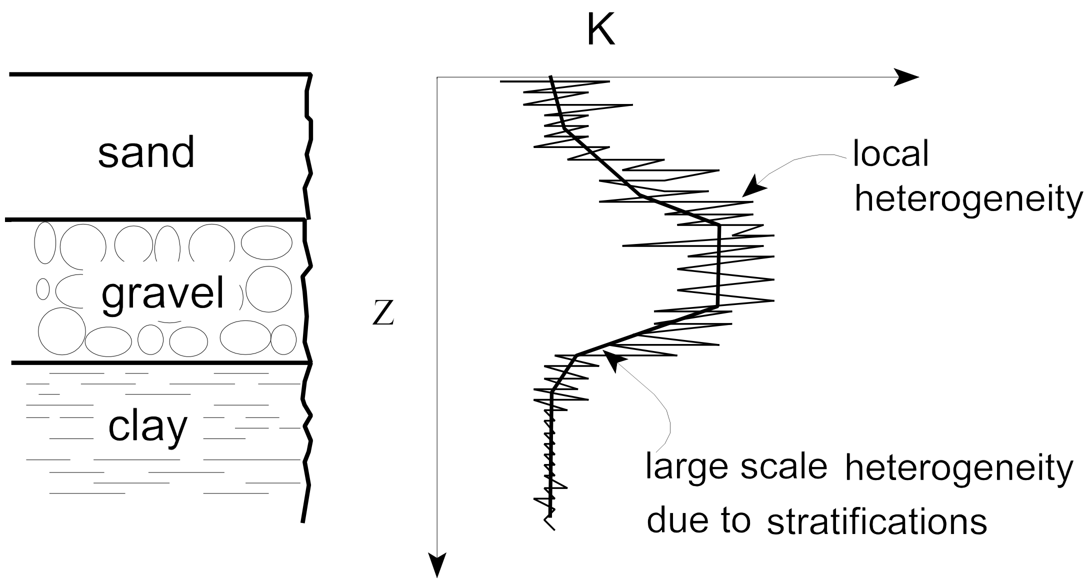
The classical ("deterministic") conceptual model, commonly used by practitioners in hydrology, assumes that Darcy's Law, developed for the laboratory-scale problem, is valid for the field-scale problem. Based on this assumption, an equivalent homogeneous and a heterogeneous approach have emerged. The first approach assumes that a heterogeneous geological medium can be treated as an equivalent homogeneous one with fictitious hydraulic properties that are constant in space. This constant hydraulic property assumption is necessary mainly due to the limitation of our ability to derive analytical solution for flow and solute transport in heterogeneous media. This approach is analogous to the REV approach for the laboratory-scale problem. First, a CV is scaled up to include heterogeneities at many different scales in the field problem. If using the CV to define a hydraulic property at various locations of the field scale problem and the property is constant everywhere, the geological medium of the field scale problem thus can be conceptualized as a homogeneous medium and the REV exists. The size of the REV can be equal to or much larger than the REV defined for the laboratory scale problem, depending on the scale of heterogeneity in the field problem. We will refer to this REV as the field-scale REV (FSREV). Traditionally, the fictitious hydraulic properties are obtained by conducting large-scale hydraulic tests and then by applying inverse approaches to identify their values (such as the classical aquifer test). Alternatively, one may average values from many small-scale tests to represent the fictitious properties (i.e., the arithmetic, geometric, or harmonic mean of conductivity values obtained from core samples). These fictitious properties are then used as input parameters to mathematical models for predicting water flow or transport of contaminants in the field. They are usually referred to as effective hydraulic properties.

The heterogeneous approach, on the contrary, visualizes the field as a collection of many elements with different hydraulic properties. In other words, we cannot define CVs that can smooth out the erratic heterogeneity to obtain a constant property everywhere in the geological formation of the field scale problem, i.e., FSREV does not exist. In this case hydraulic properties of the geological formation in the field problem are spatial variables, and they are specified spatially by using available hydrogeological information. For instance, groundwater hydrologists may use well logs and geological information to delineate large-scale geological features such as layering, stratifications, formations and fault zones. Hydraulic properties measured at some locations or reported in literature for similar geological materials are then assigned to each element by interpolation or extrapolation. This approach aims at the behavior of water flow or transport of contaminants at higher resolutions than the homogeneous approach. Similar to the laboratory-scale problem, this approach employs the CV concept but does not invoke the FSREV assumption for the entire field.

Treatment of Large-scale Heterogeneous Aquifers

1. *Heterogeneous Approach*:

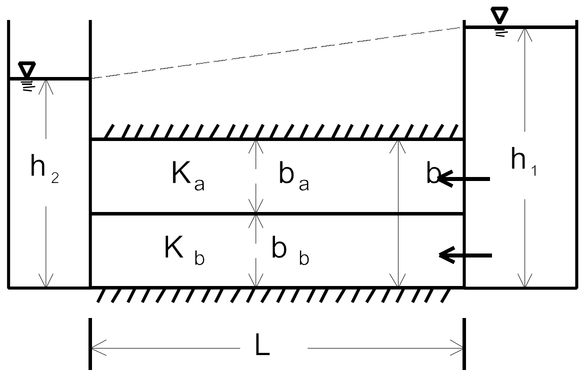
An approach to the heterogeneity problem commonly used by practitioners is to delineate all the large-scale geological formations and assign different hydraulic properties to each formation. Within each geological formation the hydraulic property is assumed to be homogeneous. Mathematical models are then used to solve the flow and transport problems.



2. *Equivalent Homogeneous Approach*:

Instead of delineating the detailed heterogeneity, this approach homogenizes the aquifer by replacing it with an equivalent homogenous one. Based on this approach, one must define equivalent homogeneous hydraulic properties (for example, effective hydraulic conductivity) for the heterogeneous aquifer. The advantage of this approach is that it avoids difficulties in the mathematical analysis of flow through heterogeneous media. To define the effective hydraulic properties, we will examine several different cases below. In these cases, we assume there exist FSREV. That is we assume the heterogeneity within the FSREV repeat itself in space of the geological formation such that the entire formation can be visualize as a homogeneous medium

Case 1. Flow direction is parallel to the stratifications



Total discharge (per unit width normal to the section) = the sum of the discharges through each layer:

$$Q = -b_a K_a \frac{(h_2 - h_1)}{L} - b_b K_b \frac{(h_2 - h_1)}{L} \qquad (1)$$

Effective hydraulic conductivity (Equivalent homogeneous K): large REV

$$Q = -b\overline{K}\frac{(h_2 - h_1)}{L} \qquad \text{where} \quad b = b_a + b_b \qquad (2)$$

Equating the quantities on the right sides of (1) and (2) yields

$$\boxed{\overline{K} = \frac{K_a b_a + K_b b_b}{b}}$$

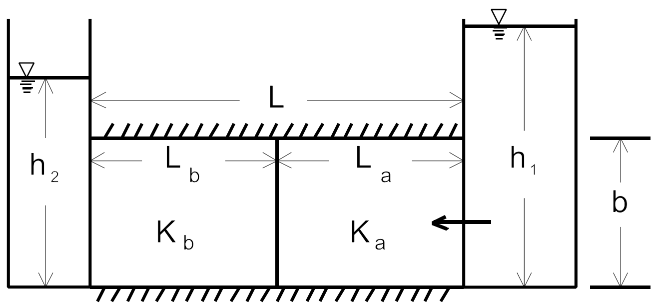
This is the effective hydraulic conductivity. Note that under the same boundary conditions (h_1 and h_2 specified) and the effective hydraulic conductivity, this equivalent homogeneous aquifer should produce the same Q as the heterogeneous one. This effective hydraulic conductivity formula can also be generalized for n layers:

$$\boxed{\overline{K} = \frac{\sum_{i=1}^{n} b_i K_i}{\sum_{i=1}^{n} b_i}} \qquad \text{Weighted Arithmetic Mean}$$

If the values for b_i are the same for each ith stratum, the effective conductivity is:

$$\overline{K} = \frac{\sum_{i=1}^{n} K_i}{n}$$ Arithmetic Mean

Case 2. Flow $\perp$ (normal to) the stratifications



Principle: Total head loss = head loss in block *a* + head loss in block *b*.

$$\Delta h = \Delta h_a + \Delta h_b$$

Now, we will find the equivalent homogeneous conductivity that produces the same total head loss with the same *Q*. Based on Darcy's Law, we can express the above head losses as

$$\frac{QL}{b\overline{K}} = \frac{QL_a}{bK_a} + \frac{QL_b}{bK_b}$$

where $L = L_a + L_b$. $\overline{K}$ (K bar) is the effective hydraulic conductivity and can be written as

$$\overline{K} = \frac{L}{\dfrac{L_a}{K_a} + \dfrac{L_b}{K_b}}$$

For *n* layers, the effective conductivity becomes

$$\overline{K} = \frac{\sum_{i=1}^{n} L_i}{\sum_{i=1}^{n} \frac{L_i}{K_i}} \qquad \text{Weighted Harmonic Mean}$$

If $L_1 = L_2 = \ldots = L_n$, then

$$\overline{K} = \frac{n}{\sum_{i=1}^{n} \frac{1}{K_1}} \qquad \text{Harmonic Mean}$$

Remarks:

From the above analysis, we can say the effective hydraulic conductivity is the conductivity of a fictitious homogeneous porous medium that produces the same discharge as that of the heterogeneous porous medium under the same boundary conditions. From this analysis, we also learn that the effective hydraulic conductivity depends on the following:

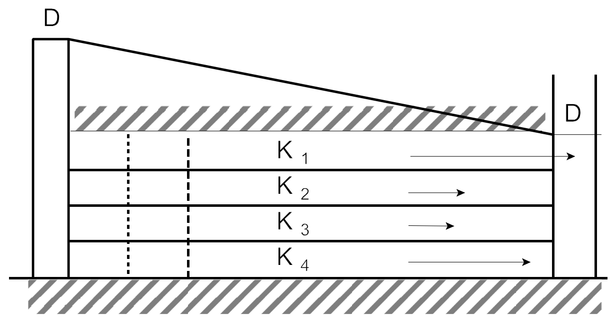
(1) Hydraulic conductivities at subscale -- the variation of the conductivity σ_f^2, $(K_1, K_2, \ldots, K_n)$.
(2) Size of Heterogeneity (thickness of layers) -- correlation scales.
(3) Direction of flow: perpendicular to bedding or parallel to bedding.

What does the effective conductivity predict?

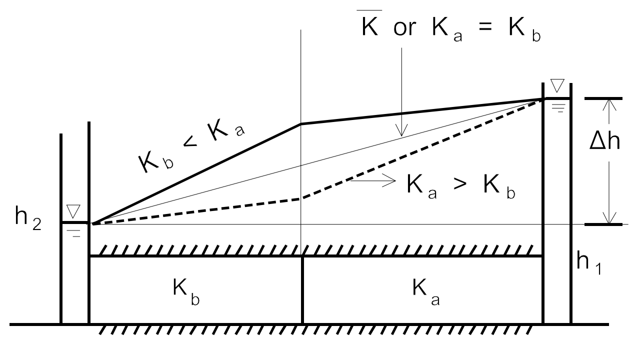
The effective hydraulic conductivity was aimed to reproduce the discharge in the heterogeneous aquifer. If one uses the effective property to determine the hydraulic response (i.e., hydraulic head) of a heterogeneous porous media, then one should ask the question: what does the calculated hydraulic head present? Let's revisit the two cases.

Head Distribution

Case 1.



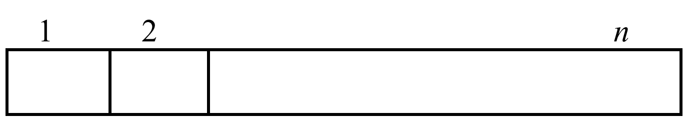
Case 2:



The straight line between the two heads represents the head distribution resulting from the use of the effective conductivity. This head distribution certainly will be different from that based on the heterogeneous one, which will depends on the magnitude of K_a and K_b (if $K_a > K_b$ or $K_a < K_b$). Thus, the straight line represents the average of the head distributions of two possible scenarios ($K_a > K_b$ or $K_a < K_b$). In other words, it is an average of all the possible head distributions resulting from all possible permutations of *K'*s. This type of average is called an *ensemble average*. Notice that the head drop and flux are the same for these two possible permutations.

Ensemble average:

If we have *n* hydraulic conductivity values, say $K_1 \ldots\ldots K_n$,

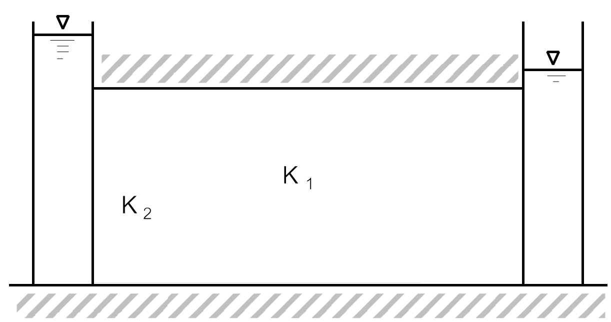


then the total number of possible permutations of the conductivity values is $n!$ $\{n\ factorial,\ (n \times (n-1) \times \ldots \times 1)\}$. As a result, one will have $n!$ possible head distributions under the same boundary conditions. If one averages all these possible head distributions along the flow path, one obtains an average head distribution, which should be equivalent to the head distribution derived from the effective hydraulic conductivity.

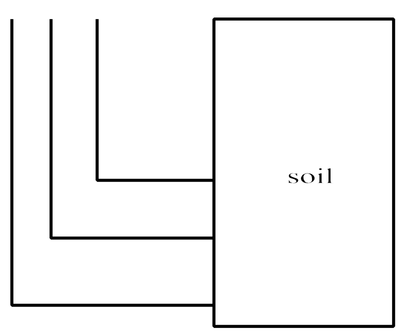
What is the effective hydraulic conductivity for a more complex heterogeneity?

For the two cases described above, involving perfectly stratified formations, the effective hydraulic conductivity can be easily defined. However, does effective *K* exist for more general cases where flow is not perpendicular or parallel to bedding or the bedding is not continuous?

What is the effective *K* $\left(\overline{K}\right)$ for the case shown in the figure?



Do you expect that the *K* determined from a soil column predicts head distribution in the permeameter or head measured in a pore? Why is it critical to use many manometers to determine the hydraulic conductivity in a soil column?

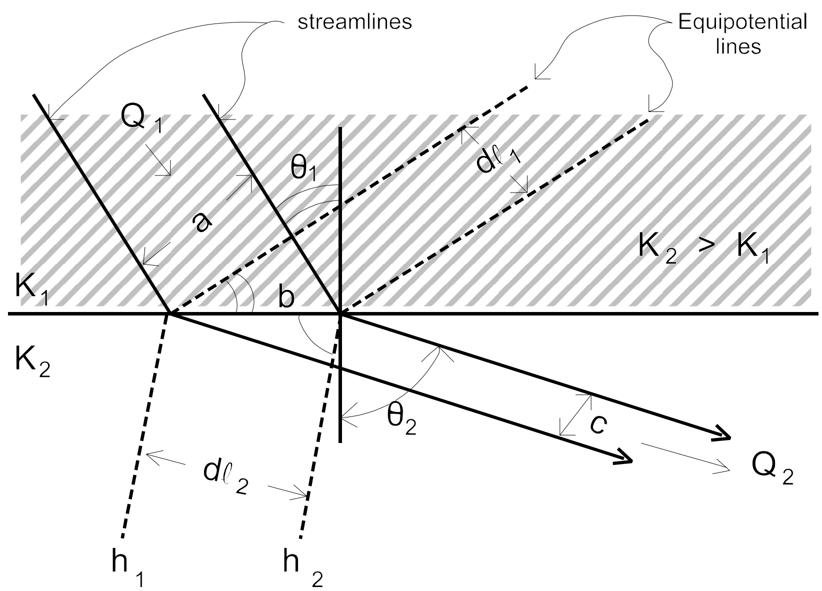


Note that the effective hydraulic conductivities are derived for steady flow. Does it exist in the transient or unsteady flow?

ANISOTROPY

Do you notice that the effective hydraulic conductivity in the direction parallel to bedding is much greater than that in the direction perpendicular to bedding? How would this phenomenon affect flow if the flow inclines to the bedding at an angle? This is the topic to be explored next.

Case 3. Flow inclined to bedding



Since the flow is steady, $Q_1 = Q_2$

$$K_1 a \frac{dh_1}{d\ell_1} = K_2 c \frac{dh_2}{d\ell_2}$$

using the relationships:

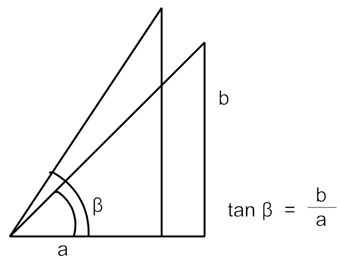
$$dh_1 = dh_2$$
$$a = b \cos \theta_1$$
$$c = b \cos \theta_2$$

$$\frac{b}{d\ell_1} = \frac{1}{\sin\theta_1} \quad and \quad \frac{b}{d\ell_2} = \frac{1}{\sin\theta_2}$$

$$\boxed{\frac{K_1}{K_2} = \frac{\tan\theta_1}{\tan\theta_2}}$$

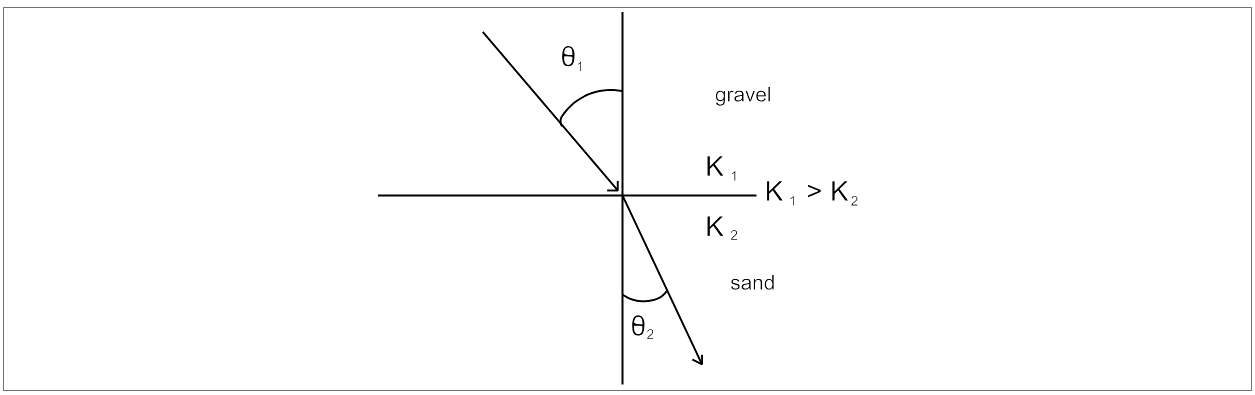
Tangent Law

since $\tan x \propto x$ large $x \rightarrow$ large $\tan x$



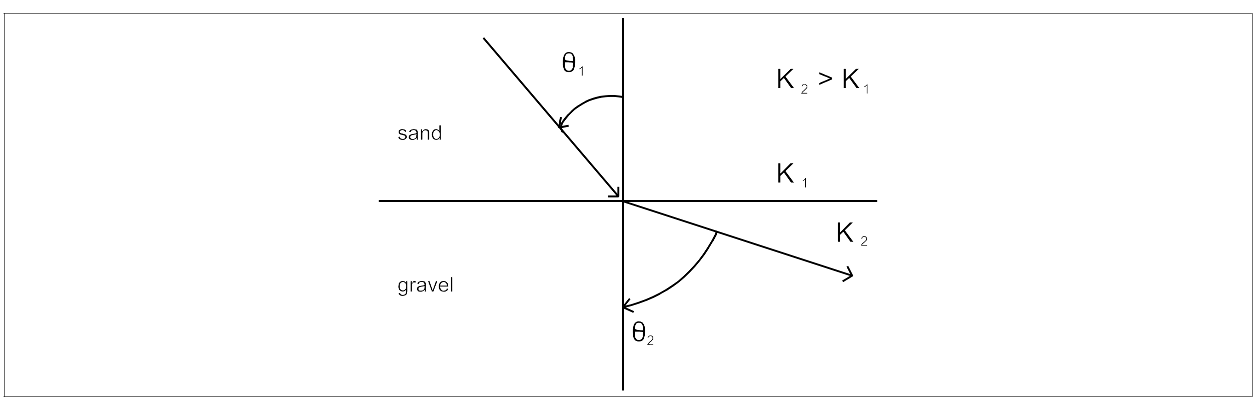
Case (1) $K_1 > K_2$ θ_1 (incident angle) is fixed

$$\tan\theta_2 < \tan\theta_1$$
$$\theta_1 > \theta_2$$



Case (2) $K_1 < K_2$ θ_1 is fixed

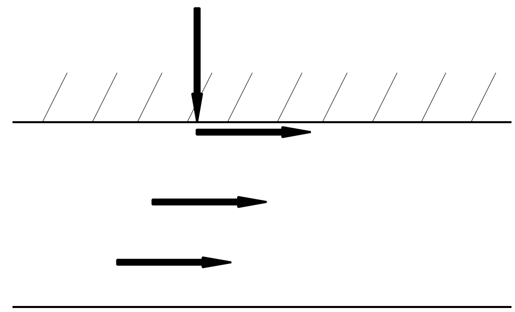
$$\tan\theta_2 > \tan\theta_2$$
$$\theta_1 < \theta_2$$



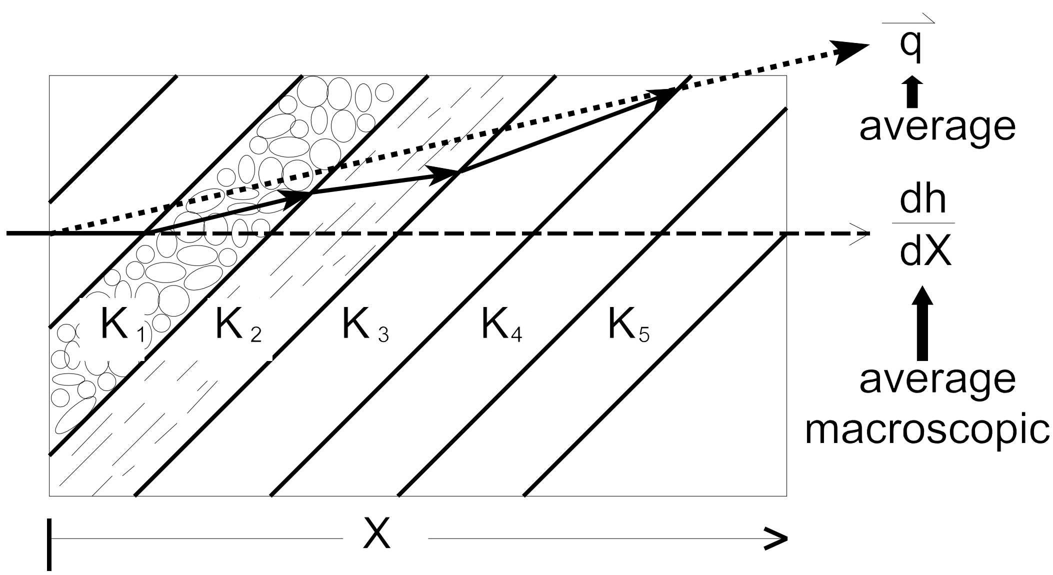
Conclusions

1. Water tries to travel through low-permeable layers by the shortest route and prefers to use high permeable layers as conduits.

2. Implications on the flow regime in leaky aquifers



Flow directions in a series of layers



According to the figure above, one would conclude that the direction of groundwater does not necessarily align with the direction of gradient on the macroscopic scale. Then, how can this fact be incorporated in the effective K?

§ ***Anisotropy & Isotropy* –> Macroscopic Property**

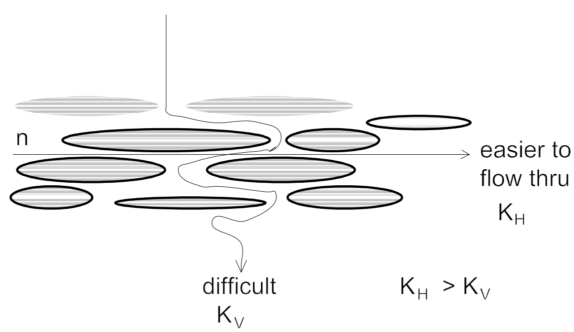
If a porous medium has a higher hydraulic conductivity in one direction (usually, the horizontal one) than in other directions, the porous medium is called **anisotropic.** On the other hand, an **isotropic** medium means that the hydraulic conductivity values of the medium are the same in all directions.

Causes of Anisotropy

The reason that anisotropy exists in our measurements of the hydraulic conductivity of geological media is again due to our use of C.V. (or volume averaging) and the presence of heterogeneities at scales smaller than the C.V. Therefore, anisotropy is also scale-dependent: it depends on the scale of the control volume and the scale and type of heterogeneity in the control volume. After all, one must recognize that anisotropy is an artifact due to our ignorance of heterogeneity at scales smaller than the CV. This means that if heterogeneity at all scales is defined, the anisotropic property of the conductivity is not necessary. On the other hand, one must recognize that one can not specify the heterogeneity at all scales and therefore, anisotropy is a useful approach.

Pore-scale Anisotropy

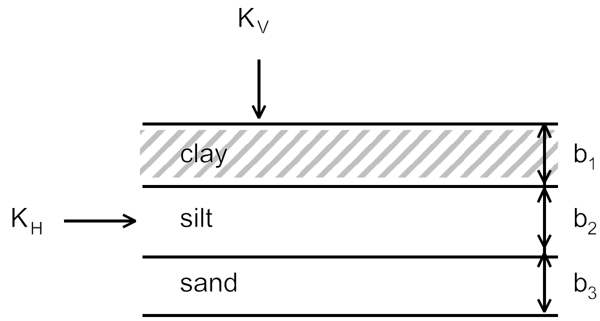
Pore-scale anisotropy arises from the fact that we measure the average conductivity over a volume of soil (a soil core). Sedimentation and pressure of overlying materials cause flat particles (minerals) to orient with the longest dimension parallel to the plane on which they settle. This produces flow channels parallel to the bedding plane, which are different from those oriented normal to this plane.



What about if the soil consists of all well-rounded particles? Any anisotropy?

Field-Scale Anisotropy

Field-scale anisotropy arises from the fact that when we measure hydraulic conductivity in a field situation, we always employ a very large R.E.V. to obtain “one” hydraulic conductivity value. However, such a large REV will likely include many large-scale heterogeneities (such as stratifications, cross-bedding, clay lenses . . . etc.)



Although within each layer the medium can be considered homogeneous and isotropic, the effective conductivity, which is an average over many conductivities of different layers, becomes anisotropic. Based on the analysis of Cases 1 and 2, if we visualize these layers as an "equivalent Homogeneous" formation, then

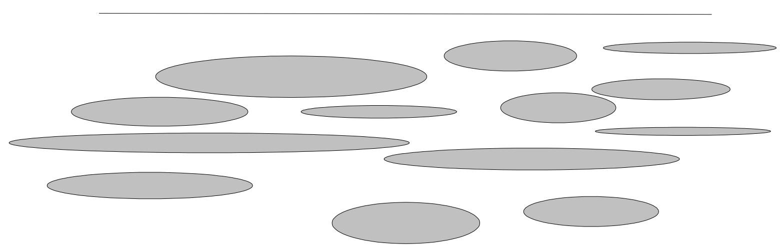
K_V = Harmonic mean if $b_1 = b_2 = b_3$

K_H = Arithmetic mean if $b_1 = b_2 = b_3$

Anisotropy = $\boxed{\frac{K_H}{K_V} > 1}$

Questions:

1. What is the anisotropy if the layer is not perfectly stratified? How would this anistropy be related to the average dimension of layers and the harmonic and arithmetic means?



What if the layer thickness varies with distance, are the harmonic and arithmetic averages valid? Are the effective hydraulic conductivity the same if the flow direction changes?

Classification of Porous Media

1. Homogeneous isotropic $K(x, y, z) = K$
2. Heterogeneous isotropic $K(x, y, z) \neq K$
3. Homogeneous anisotropic K_x, K_y, K_z
4. Heterogeneous anisotropic $K_x(x, y, z), K_y(x, y, z), K_z(x, y, z)$

Heterogeneity and Anisotropy in Unsaturated Porous Media

1. Can a heterogeneous porous medium behave as a homogeneous medium under some situations?

2. Can you carry out the same analysis as in the saturated flow to derive the effective hydraulic conductivity for unsaturated porous media?

3. What do you think about the anisotropy in the unsaturated medium? Does it vary with moisture content? (See Yeh et al., 1985 WRR).