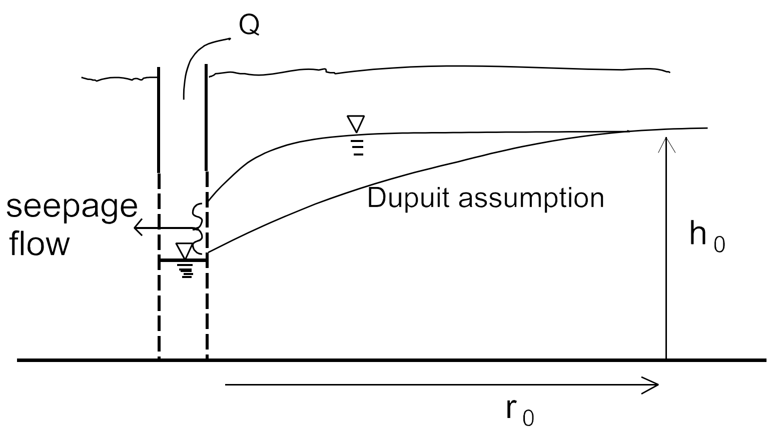
§ **UNCONFINED AQUIFER** (W.T. or Phreatic Aq.) 535-5e1.wpd

1. Steady flow

Assumptions:

1. Isotropy and homogeneity
2. const. discharge Q
3. horizontal bed
4. Dupuit assumptions:
   a. horizontal flow
   b. no seepage face above the water table



Governing P.D.E. (plan view)

$$K\frac{\partial^2 h^2}{\partial x^2} + K\frac{\partial^2 h^2}{\partial y^2} = 0$$

Depth-averaged head

Polar coordinates

$$\frac{d^2 h^2}{dr^2} + \frac{1}{r}\frac{dh^2}{dr} = 0$$

$$h(r_0) = h_0 \qquad \text{at } r = r_0$$

$$\lim_{r \to 0} r\frac{dh}{dr} = \frac{Q}{2\pi Kh}$$

B.C.

Solution

$$h_0^2 - h^2 = \frac{Q}{\pi K}\ln\left(\frac{r_0}{r}\right)$$

Nonlinear in terms of h but linear in terms of h^2

Note: Because of the underlying Dupuit assumptions, the solution fails to describe the hydraulic head distribution near the well. (Horizontal flow & seepage face)

$$K = \frac{Q}{\pi\left(h_2^2 - h_1^2\right)}\ln\left(\frac{r_2}{r_2}\right)$$

This equation is limited to locations far away from the pumped well only.

§ Jacob's Correction Method for Water Table Drawdown

$$h = h_0 - s$$

From (1) using two observation wells

$$Q = \frac{\pi K\left(h_2^2 - h_1^2\right)}{\ln\left(r_2/r_1\right)}$$

$$h_1^2 = h_0^2 - 2h_0 s_1 + s_1^2$$

$$h_2^2 = h_0^2 - 2h_0 s_2 + s_2^2$$

$$h_2^2 - h_1^2 = 2h_0\left[\left(s_1 - \frac{s_1^2}{2h_0}\right) - \left(s_2 - \frac{s_2^2}{2h_0}\right)\right]$$

Let $s_c = s - \dfrac{s^2}{2h_0}$ $\rightarrow$ corrected drawdown

$$Q = \frac{2\pi K\left(s_{c_1} - s_{c_2}\right)}{\ln\left(r_2/r_1\right)}$$

linear, use superposition for various cases

<u>§ Transient Unconfined Flow</u> (Jacob, U.S.G.S. Water Supply Paper 1536I)

1. Use Dupuit Assumptions, homogeneity & isotropy

$$\frac{1}{r}\frac{\partial}{\partial r}\left(rKh\frac{\partial h}{\partial r}\right) = S_y \frac{\partial h}{\partial t} \qquad (1)$$

or

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial h^2}{\partial r}\right) = \frac{S_y}{Kh}\frac{\partial h^2}{\partial t} \qquad (2)$$

Now we let $s = h_0 - h$ or $h = h_0 - s$

where h_0 is the initial hydraulic head

$$\frac{\partial h^2}{\partial r} = \frac{\partial}{\partial r}\left(h_0 - s\right)^2 = \frac{\partial}{\partial r}\left(h_0^{\ 2} - 2sh_0 + s^2\right)$$

$$= -2h_0 \frac{\partial}{\partial r}\left(s - \frac{s^2}{2h_0}\right)$$

$$= -2h_0 \frac{\partial s_c}{\partial r}$$

where

$$s_c = s - \frac{s^2}{2h_0}$$

Similarly

$$\frac{\partial h^2}{\partial t} = -2h_0 \frac{\partial s_c}{\partial t}$$

Eq, (2) becomes

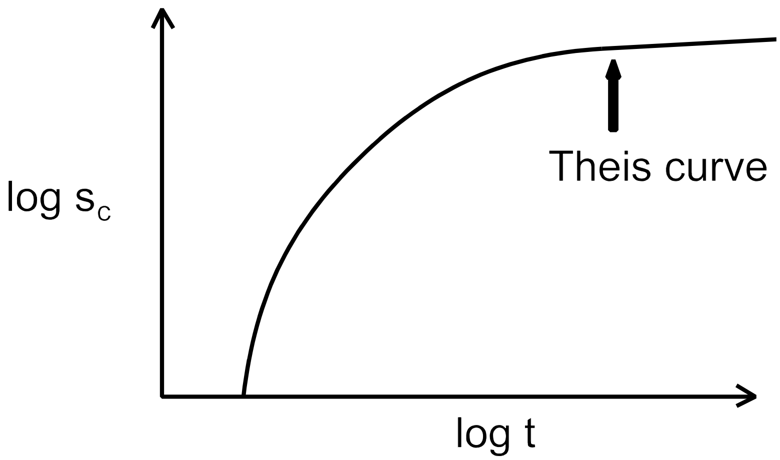
$$\frac{1}{r}\frac{\partial}{\partial r}\left(r(-2h_0)\frac{\partial s_c}{\partial r}\right) = \frac{S_y}{K(h_0 - s)}(-2h_0)\frac{\partial s_c}{\partial t}$$

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial s_c}{\partial r}\right) = \frac{S_y}{T}\frac{\partial s_c}{\partial t}$$

$$T(r,t) = K \cdot h(r,t)$$

if we assume $h >> s$ $\quad T(r, t) \approx$ constant. This P.D.E. is identical to that of confined flow.

⇒ The solution to this P.D.E. will conform with the Theis solutions, i.e.



Since $h >> s \Rightarrow s_c = s - \frac{s^2}{2h_0} \approx s$

Conclusions:

1. The drawdown-time curve of flow to a well in an unconfined Aq. should be similar to that of confined flow.

   i.e. → Theis' Curve

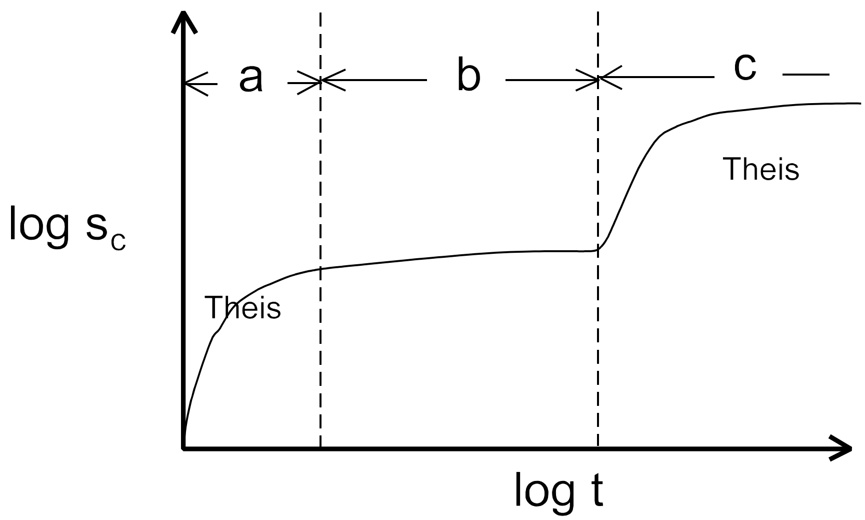
2. But different Mechanisms:

   a. In confined aquifers water is released by:
      (1) expansion of water
      (2) compaction of porous media

   b. In unconfined aquifers, water is mainly released by gravity drainage (dewatering process)

3. The reason that s_c - t data conform to the Theis Curve is due to the assumption of horizontal flow.

Field data → S shape drawdown-time data are often observed



Three distinct segments:

A. Early time

1. drawdown behaves in a way similar to that in a confined aquifer

2. water table does not fall instantaneously

B. Intermediate time

1. drawdown-time data deviate from Theis' Curve

2. water table starts falling noticeably

3. drawdown rate slows down

drawdown-time data similar to those in leaky aquifers

4. water level in the fully penetrating O.B. well is much lower than the W.T.

C. Late time

1. W.T. coincides with the water level in the O.B. well and falls at the same rate as the drawdown

2. drawdown-time data again conform closely to the Theis Curve

WHY ?

Boulton (1954, 1963) & Prickette (1965) introduced a "Delayed Yield Response" concept.

$$\frac{\partial^2 s}{\partial r^2} + \frac{1}{r}\frac{\partial s}{\partial r} = \frac{S}{T}\frac{\partial s}{\partial t} + \alpha\frac{S_y}{T}\int_0^t \frac{\partial s}{\partial t} e^{-\alpha(t-\gamma)} d\gamma$$

↓ ↓

ag. elastic effect Delayed yield

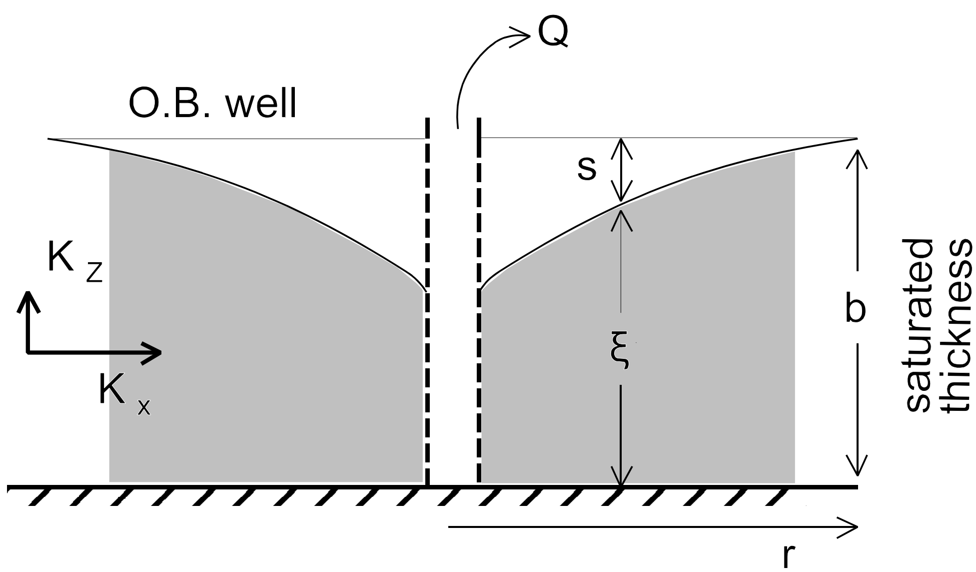
delay yield index → $1/\alpha$ (empirical constant)

Delay yield → an additional source who's contribution increases with time.

Remarks:

1. fits the data
2. Lack of physical basis
3. α fudge factor

§ Neuman's Analysis (1972, 1973b, 1975b, 1979) [Reading Assignments]



Assumptions:

1. unconfined aquifers of infinite lateral extent
2. impermeable horizontal layer (bottom)
3. Homogeneous, but could be isotropic or anisotropic. principal *K*s are aligned with the *r*, *Z* coordinates
4. complete penetrating well with a constant *Q*

Governing P.D.E.

$$K_r \frac{\partial^2 s}{\partial r^2} + \frac{K_r}{r}\frac{\partial s}{\partial r} + K_z \frac{\partial^2 s}{\partial Z^2} = S_s \frac{\partial s}{\partial t}$$

I.C. $s(r, Z, 0) = 0$ $\rightarrow$ drawdown at a point

$\xi(r,0) = b$

B.C. $s(\infty, Z, t) = 0$

$$\frac{\partial s}{\partial Z}(r,0,t) = 0 \qquad \rightarrow \text{impermeable B.C.}$$

Another boundary condition:

Free-surface boundary condition for the water table.

$$K_r \frac{\partial s}{\partial r} n_r + K_Z \frac{\partial s}{\partial Z} n_Z = \left(S_y \frac{\partial \xi}{\partial t} - I \right) n_Z$$

at (r, ξ, t)

where $\xi(r,t) = b - s(r, \xi, t)$

$$\lim_{r \to 0} \int_0^{\xi} r \frac{\partial s}{\partial r} dr = + \frac{Q}{2\pi K r}$$

Notice that no depth-averaging (or Dupuit assumption) is involved and it allows vertical flow in the flow regime.

Solutions $\Rightarrow$ complicated (see Neuman, 1972)

If the observation well is perforated through the entire saturated thickness, the drawdown is

$$\bar{s}(r,t) = \frac{1}{b}\int s(r,z,t)dz$$

which is the depth average drawdown and Neuman's solution gives

$$\bar{s}(r,t) = \frac{Q}{4\pi T} W\left(\mu_a, \mu_a, \eta\right)$$

where $W\left(\mu_a, \mu_a, \eta\right)$ is known as an unconfined well function (shown in handout)

$$\eta = \frac{r^2 K_z}{b^2 K_r}$$

If $S_y >> S$ and $\bar{s} << b$

At early time

$$\bar{s} = \frac{Q}{4\pi T} W\left(u_a, \eta\right) \qquad \text{(Similar to } \frac{r}{B} \text{ solution)}$$

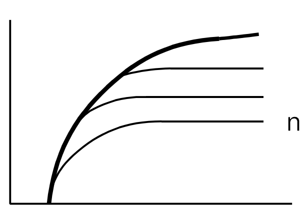
where S is the storage cavity

→ expansion of water

→ compaction of porous media

→ Horizontal flow (at very early time values)

→ conform to Theis Curve (at very early time values)



At later time

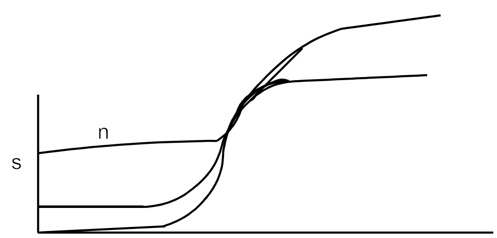
$$\bar{s} = \frac{Q}{4\pi T} W\left(u_b, \eta\right)$$

$$u_b = \frac{r^2 S_y}{4Tt}$$

where S_y is specific yield

$\Rightarrow$ gravity drainage (dewatering)

$\Rightarrow$ horizontal flow (Theis)



Type Curve $\Rightarrow$ see handout.

At Intermittence

vertical flow dominates

$\rightarrow$ leaky aquifer

$\rightarrow$ drawdown stabilizing

§ Determination of T, S, S_y, K_r, K_Z

Neuman 1975, "Analysis of Pumping Test Data from Anisotropic Unconfined Aquifer Considering Delayed Gravity Response", *WRR*, 11(2), pp. 299-342.

A. Type Curve Method

1) plot s - t curve on log-log paper

2) match as much of the latest field drawdown-time data to one of the type B curves; determine:

s^*, t^*, W^*, and $1/u_B^*$ of the match point (m.p) and η_B

3) Determine T and S_y

$$T_B = \frac{QW^*}{4\pi s^*}$$

$$S_y = \frac{4Ttu_B^*}{r^2}$$

4) Match as much of the earliest s - t data to one of the type A curves; find s, t and $W, 1/u_A$ of the m.p and η again

$$\eta_A \approx \eta_B = \eta$$

5) determine T & S

$$T_A = \frac{QW^*}{4\pi s^*} \qquad T_A \approx T_B$$

$$S = \frac{4Tt^* u_A^*}{r^2} \qquad s << b$$

6) Horizontal hydraulic conductivity

$$K_r = T/b$$

where b is the thickness of the saturated zone

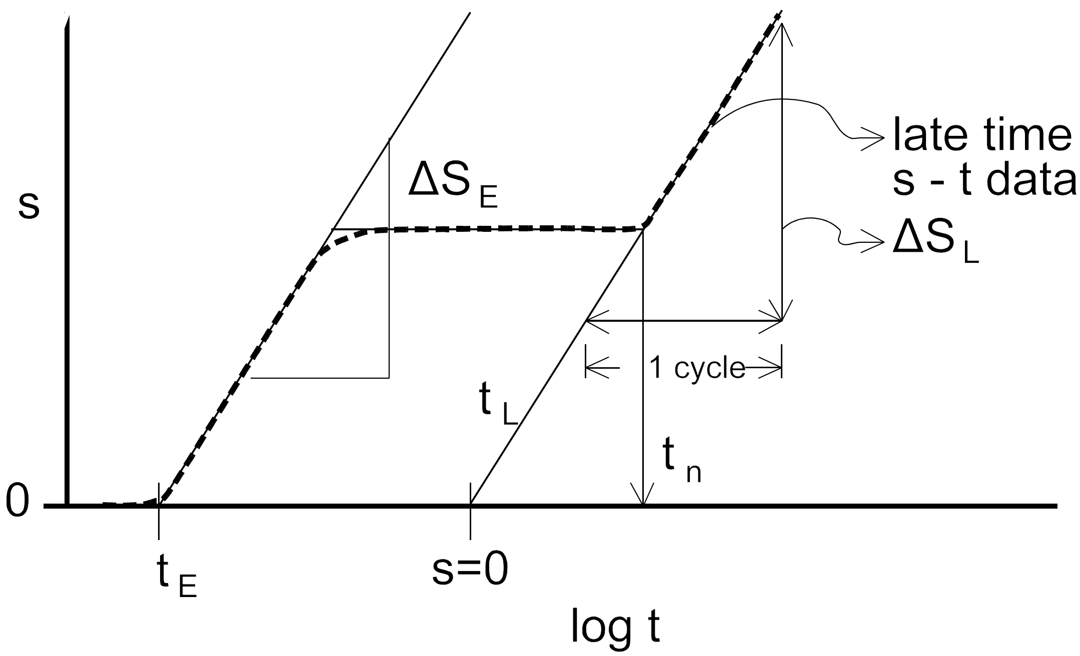
7) Vertical hydraulic conductivity K_Z

$$\eta = \frac{r^2 K_Z}{b^2 K_r}$$

$$K_Z = \frac{\eta b^2 K_r}{r^2}$$

B. Semi-logarithmic Method

1) plot A vs. $\log t$



2) use late time data

a. find t_L at $s = 0$

b. take 1 log cycle on t axis $\Rightarrow$ ΔS_L

$$T = 0.183\left(Q/\Delta S_L\right)$$

$$S_Y = 2.25\frac{Tt_L}{r^2}$$

3) A horizontal line is fitted to the intermediate portion of the s - t data

a. find t_η (t coordinate of the intersection of the horizontal line and the late time data line)

$$t_{y\eta} = \frac{4Tt\eta}{S_y r^2}$$

if $4.0 \leq t_{y\eta} \leq 100.0$

$$\eta \approx \frac{0.195}{t_{y\eta}^{1.1053}}$$

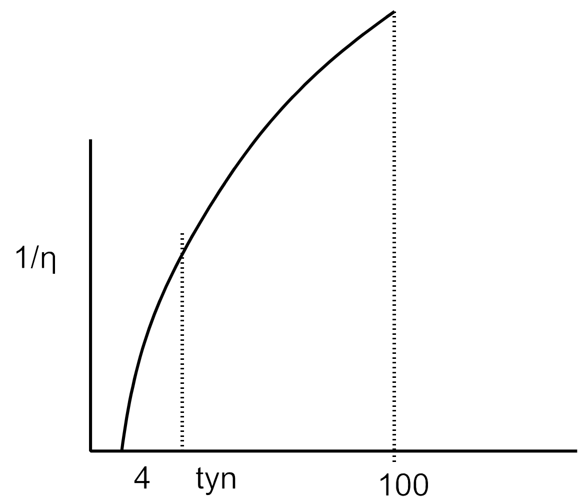
b. otherwise use Figure 3. (Neuman, 1975) to determine η

4) A straight line is fitted to a portion of the early time-drawdown data

1. If the slope of this line $\neq$ that of the line in step 2;
   $u \geq 0.01$; use this curve $\rightarrow S$

2. They are parallel

   a. find t_E at $s = 0$



   b. take 1 log cycle on t axis

$$\Rightarrow \Delta S_E$$

$$T = \frac{0.183Q}{\Delta S_E}$$

$$S = \frac{2.25Tt_E}{r^2}$$

5) use η and follow steps 6 and 7 in Type curve method to determine:

$$K_Z, K_r$$