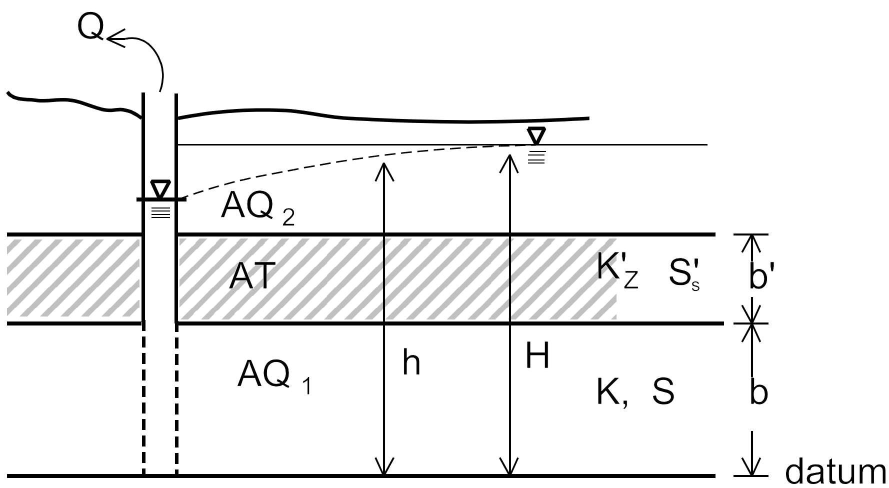
§ LEAKY CONFINED AQUIFER

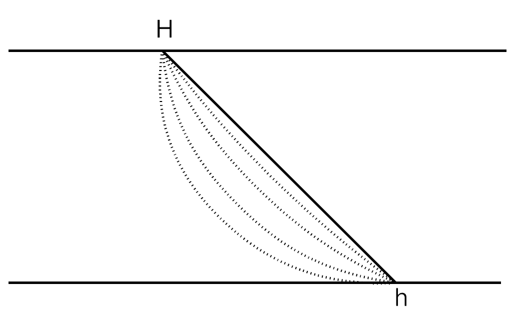
There are several analytical solutions developed in the past for leaky confined aquifers. Each solution relies on some simplifications of true flow systems. First, we will introduce a solution that does not consider the effect of storage in the aquitard, and then a solution that considers the storage effect in the aquitard.

1) Without storativity in the aquitard $S'_s = 0$



Assumptions:

1) Homogeneous and isotropic aquifer with infinite lateral extent
2) No drawdown in AQ_2, implying that H = const.
3) Steady flow through AT, implying that $S'_s = 0$



4) One-dimensional vertical flow in AT to AQ_1

5) Constant pumping Q
6) Initially $h = H$

Governing Partial Differential Equation for the flow in the above leaky confined aquifer, AT_1.

$$T\left(\frac{\partial^2 h}{\partial x^2} + \frac{\partial^2 h}{\partial y^2}\right) + q_Z = S\frac{\partial h}{\partial t}$$

where q_z represents the inflow from the aquitard above the aquifer and it is defined as

$$q_Z = -K_Z^{'}\frac{(H-h)}{b'}$$

where K'_Z and b' are the vertical hydraulic conductivity and the thickness of the aquitard.

Combining the two equations, we obtain

$$\frac{\partial^2 h}{\partial x^2} + \frac{\partial^2 h}{\partial y^2} - \frac{K_Z^{'}(H-h)}{Tb'} = \frac{S}{T}\frac{\partial h}{\partial t}$$

Use the polar coordinates and let the drawdown $s = H - h$. The equation can be expressed as:

$$\frac{\partial^2 s}{\partial r^2} + \frac{1}{r}\frac{\partial s}{\partial r} - \frac{s}{B^2} = \frac{S}{T}\frac{\partial s}{\partial t}$$

in which

$$B = \sqrt{Tb'/K_Z^{'}} = [L] \quad \rightarrow \text{Leakage factor}$$

and K'_Z/b' is called leakance or leakage coefficient, which has the dimension $[1/T]$. The boundary and initial conditions associated with this problem are

B.C.

$$s(\infty, t) = 0$$

$$\lim_{r\to 0} r\frac{\partial s}{\partial r} = \frac{Q}{2\pi T}$$

I.C.

$$s(r,0)=0$$

§ Hantush - Jacob Solution $\left(\text{"}\frac{r}{B}\text{" solution}\right)$

$$s(r,t)=\frac{Q}{4\pi T}W\left(u,\frac{r}{B}\right)$$

where

$$W\left(u,\frac{r}{B}\right)=\int_u^\infty \frac{1}{y}\exp\left(-y-\frac{r^2}{4B^2y}\right)dy$$

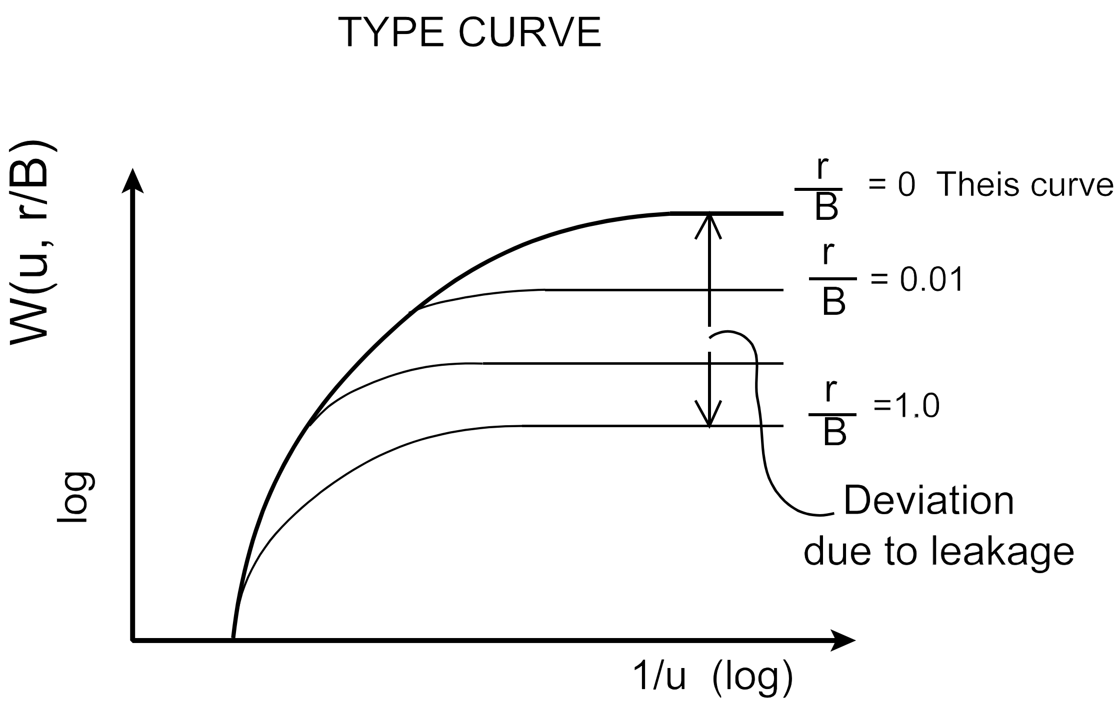
$$u=\frac{r^2S}{4Tt}$$

A numerical tabulation of $W\left(u,\frac{r}{B}\right)$ is given in Table 3.5 of the handout

$$or\ \frac{r}{B}\Rightarrow 0\Rightarrow W\left(u,\frac{r}{B}\right)\Rightarrow W(u)$$

$$\text{if}\qquad \frac{K_Z^{'}}{K}\rightarrow 0\Rightarrow B=\sqrt{Kbb'/K_Z^{'}}\rightarrow\infty$$

or r/b approaches zero and the $W(u, r/B)$ becomes $W(u)$ which is the Theis well function for non-leaky confined aquifers. The behavior of the function, $W(u, r/B)$ is shown in the figure below.



Remarks:

Based on the figure, we can conclude that

1. Leakage from the aquitard stabilizes the drawdown in the main aquifer, causing its deviation from the Theis solution for the confined aquifer. The drawdown behavior is similar to that of the case where the aquifer is in connection with a recharge boundary as we discussed earlier.

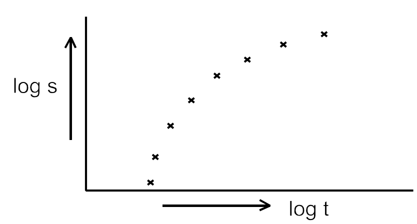
2. At a fixed r, if B is small (i.e., K/K_z' is small or the vertical conductivity of the aquitard is relatively large) the drawdown s in the main aquifer stabilizes (or reaches a steady state) fast.

3. As time goes on, more and more well discharge is derived from leakage and ultimately the entire yield of the well is due to leakage and flow becomes steady.

§ Estimation of T & S, & K'_Z

One way to estimate the parameters for this type of leaky confined aquifer is to use the type-curve matching method. Use the type curves (see handout) to match the data similar to the confined aquifer cases.

$$\text{determine} \quad u, \frac{r}{B}, W\left(u, \frac{r}{B}\right), s, t$$



To determine the vertical hydraulic conductivity (K'_z), you need to know the thickness of the aquitard, b', which is generally estimated from the well-log data.

Steady-State Flow

The governing equation becomes

$$T = \frac{Q}{4\pi s} W\left(u, \frac{r}{B}\right)$$

$$K_Z' = \frac{Tb'\left(\frac{r}{B}\right)^2}{r^2}$$

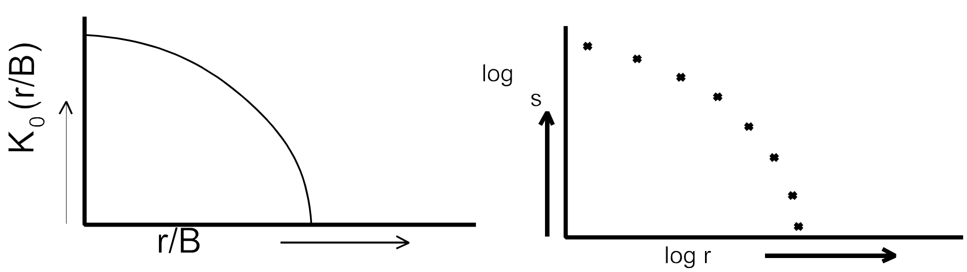
$$S = \frac{4Ttu}{r^2}$$

$$\frac{\partial^2 s}{\partial r^2} + \frac{1}{r}\frac{\partial s}{\partial r} - \frac{s}{B^2} = \frac{S}{T}\frac{\partial s}{\partial t} = 0$$

where the B.C.'s are the same as in the previous unsteady state case. Solution:

$$s(r) = \frac{Q}{2\pi T} K_0\left(\frac{r}{B}\right)$$

where K_0 is the modified Bessel function of the second kind and of zero order (The close encounter of the third kind to me). (see Table 3.6). The behavior of the drawdown as a function of distance is shown in the following figure. Type-Curve match provides a means to estimate parameter values.



The type-curve can be constructed by using Table (3.6). Parameters are obtained by using the following formulas.

$$T = \frac{Q}{2\pi s} K_0\left(\frac{r}{B}\right)$$

$$K_Z^{'} = \frac{Tb'(r/B)^2}{r^2}$$

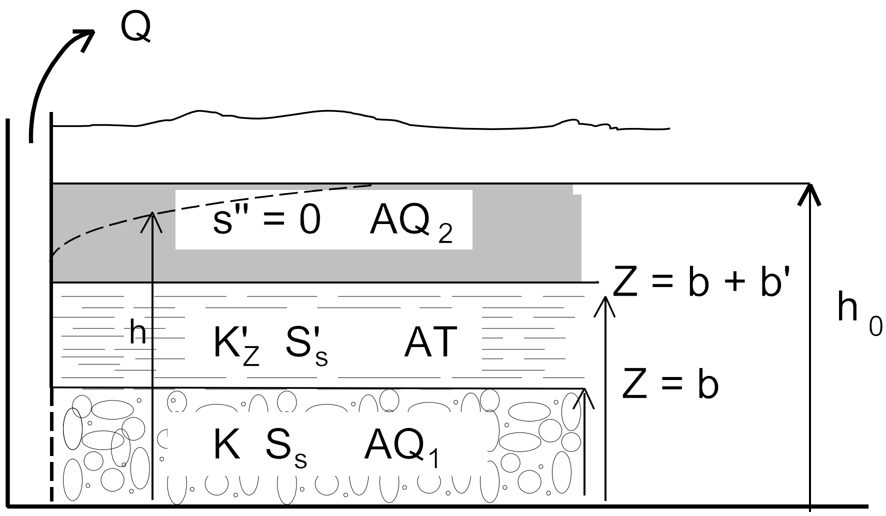
Notice that:

1. We are dealing with steady flow so that drawdown at the observation wells should not vary with time.
2. S can not be determined by this test, because the entire yield of the pumped well is derived from leakage.

(2) Leaky confined aquifers with storativity in aquitard $S_{s'} \neq 0$ (AT)

Assumptions:

(1) Homogeneous and isotropic with infinite lateral extent
(2) No drawdown in $AQ_2 \rightarrow H = \text{const.}$
(3) Vertical flow in AT
(4) const. Q



In this analysis, we allow transient (unsteady) leakage through the aquitard. That is the governing flow equation for the aquitard is

A) Aquitard

$$\frac{\partial^2 h'}{\partial Z^2} = \frac{S_s'}{K_Z'}\frac{\partial h'}{\partial t}$$

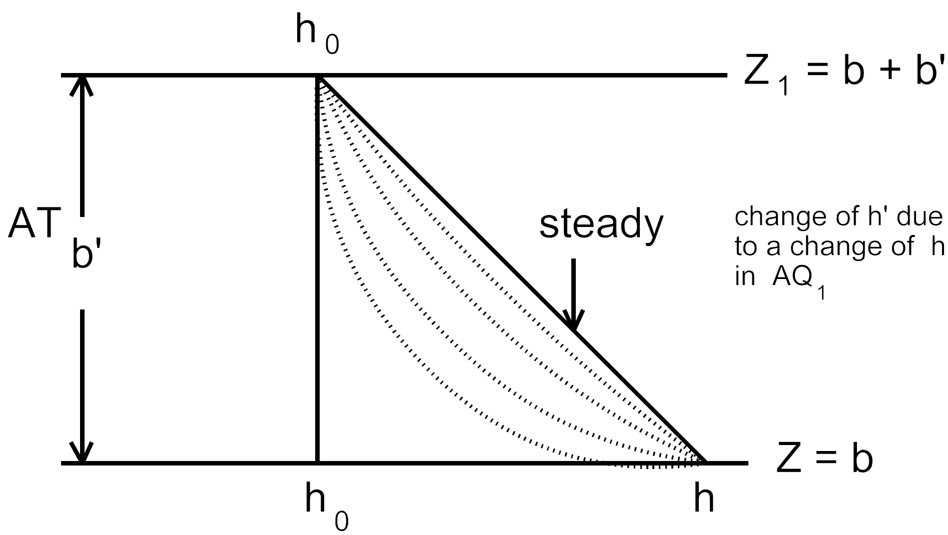
where h', K'_z, and S'_s are the head, vertical conductivity, and storativity of the aquitard, respectively. The initial and boundary conditions associated with the flow regime are

I.C. $h'(Z,0) = h_0 b < Z < b + b'$

B.C. $b'(b,t) = h(b,t)$ $\rightarrow$ Hydr. head in the AQ_1

$h'(b + b',t) = h_0$

The first boundary condition simply means that the hydraulic head of the aquitard should be equal to that of the main aquifer, AQ_1, at any time (i.e., an interface boundary condition). The general behavior of the head in the aquitard is depicted below.



Now, we need the equations for the flow in the main aquifer

B) AQ_1

$$T\left(\frac{\partial^2 h}{\partial r^2} + \frac{1}{r}\frac{\partial h}{\partial r}\right) + q_z = S\frac{\partial h}{\partial t}$$

I.C. $h(r,0) = h_0$

B.C. $h(\infty,t) = h_0$

$$\lim_{r\to 0} r\frac{\partial h}{\partial r} = +\frac{Q}{2\pi T}$$

where

$$q_Z = -K_Z^{'}\frac{\partial h'}{\partial Z}$$

↑

obtained from the solution of h' in AT

Asymptotic solutions (Hantush, 1964. Advances in Hydroscience, by Chow, Vol.1.196W)
"(β solution)"

$$t < 0.1\frac{S_s^{'}(b')^2}{K_Z^{'}}$$

(1) Early time solution

$$s(r,t) = \frac{Q}{4\pi T}\left[\int_u^\infty \frac{e^4}{y} erfc\left(\frac{\beta\sqrt{u}}{\sqrt{y(y-u)}}\right)dy\right]$$

$$= \frac{Q}{4\pi T}H(u,\beta) \qquad \text{-- see handout}$$

where

$$\beta = \frac{r}{4b}\sqrt{\frac{K_Z^{'}S_s^{'}}{KS_s}}$$

(2) Late time solution

$$t > \frac{2(b')^2 S_s'}{K_Z'} \quad \text{and} \quad t > \frac{30\delta_1 r^2 S_s}{K\left[1-\left(\frac{10r}{B}\right)^2\right]}$$

$$\text{with} \quad \frac{r}{B} < 0.1, \qquad \delta_1 = 1 + \frac{S'}{3S}$$

$$s = \frac{Q}{4\pi T} W\left(\delta, u, \frac{r}{B}\right) \qquad \rightarrow \text{use } \frac{r}{B} \text{ solution}$$

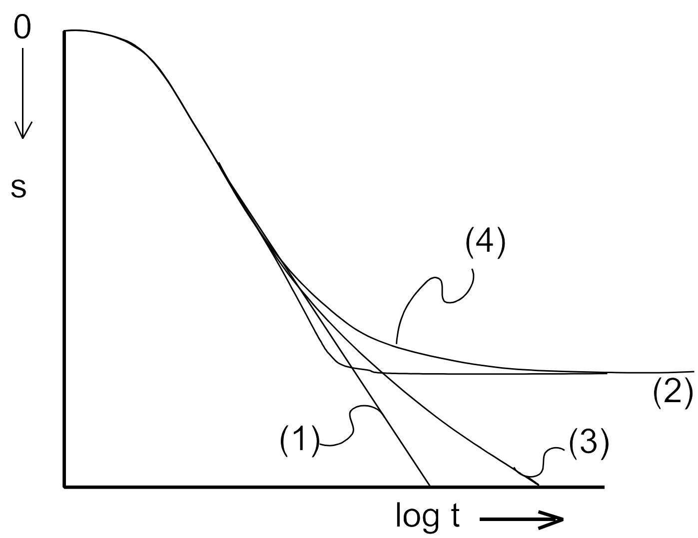
if $S' = 0$ $\quad \delta_1 = 1$

$$s = \frac{Q}{4\pi T} W\left(u, \frac{r}{B}\right)$$

K_Z' can be determined !!

As you may notice, the solutions to these equations are rather bloody. I have no interest in digging out the detail for you and you do not need it anyway. Rather, we will examine the general behavior of the flow regime in this type of aquitard-aquifer system.

Drawdown - Time Curves for Non-Leaky and Leaky Confined Aquifers with Infinite Lateral Extent.



(1) Non-leaky aquifer (infinite lateral extent)

(2) Leaky aquifer without storage in an aquitard with finite thickness
   Drawdown remains constant as soon as the discharge (well) equals the leakage from the aquitard (steady-state)

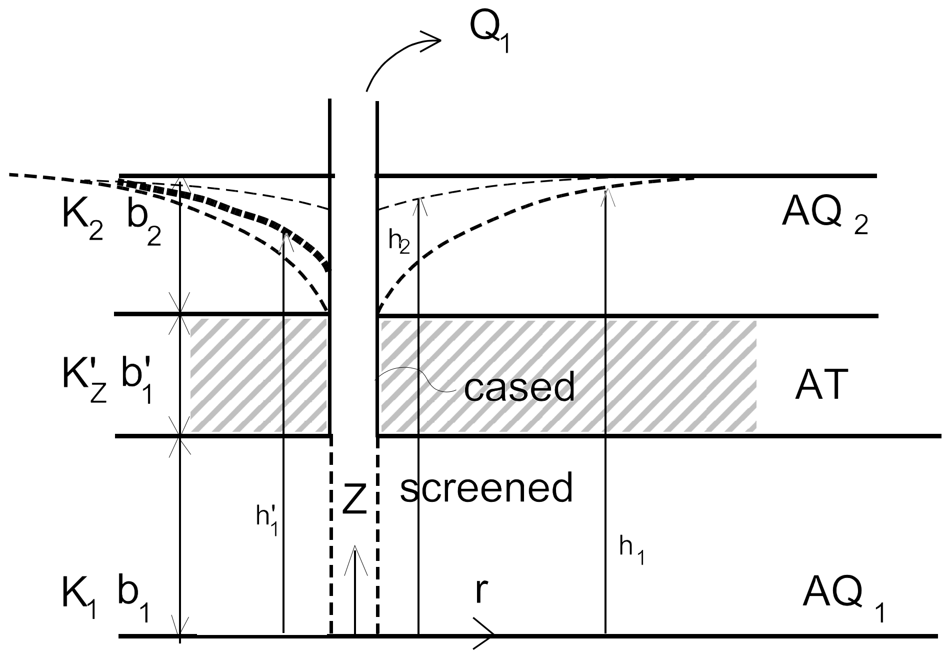
(3) Leaky aquifer with storage in an aquitard of infinite thickness
   a. No steady leakage due to infinite thickness
   b. flow in the main aquifer cannot reach a steady state

(4) Leaky aquifer with storage in an aquitard of finite thickness
   Steady-state in the aquifer will be reached when the leakage is equal to the well discharge. However, the time to steady-state is delayed by the storage effect of the aquifer. The time to the steady-state (response time) depends mainly on:

$$(1)\ K_Z \qquad (2)\ b' \qquad (3)\ S_s'$$

Neuman & Witherspoon, 1969. *Water Resources Research* 5(4), pp. 803~815

Neuman and Witherspoon derived a complete analytical solution for a more realistic flow regime in the leaky aquifer.



Their analysis allows heads in the upper aquifer and the aquitard to vary with the head in the bottom main aquifer where the screened well taps into. In order to analyze such a complex situation, one must solve three governing flow equations for each aquifer and the aquitard. To simply the analysis, they had to use the following assumptions:

(1) flow is vertical and one-dimensional in the aquitard (AT),

(2) only horizontal flows exist in both the lower and upper aquifers, namely, AQ_1, AQ_2, respectively.

Governing Partial Differential Equations for the pumped lower aquifer, AQ_1, in terms of drawdown.

$$\frac{\partial^2 s_1}{\partial r^2} + \frac{1}{r}\frac{\partial s_1}{\partial r} + \frac{K_Z^{'}}{T_1}\frac{\partial s_1^{'}}{\partial Z} = \frac{S_1}{T_1}\frac{\partial s_1}{\partial t}$$

↑

source term

recharge from AT

I.C.

$$s_1(r,0) = 0$$
$$s_1(\infty,t) = 0$$
$$\lim_{r\to 0} r\frac{\partial s_1}{\partial r} = \frac{Q}{2\pi T}$$

Equations for the aquitard, AT (Vertical flow Assumption)

$$\frac{\partial^2 s_1}{\partial z^2} = \frac{S_s}{K_z}\frac{\partial s_1}{\partial t}$$

with the following initial and boundary conditions

I.C. $s_1^{'}(r,Z,0) = 0$

B.C. $s_1^{'}(r,0,t) = s_1(r,t)$

$s_1^{'}(r,b_1^{'},t) = s_2(r,t)$

The governing equations for the upper aquifer are

AQ_2

$$\frac{\partial^2 s_2}{\partial r^2} + \frac{1}{r}\frac{\partial s_2}{\partial r} + \frac{K_1^{'}}{T_2}\frac{\partial s_1^{'}}{\partial Z} = \frac{S_2}{T_2}\frac{\partial s_2}{\partial t}$$

↑

sink term losing

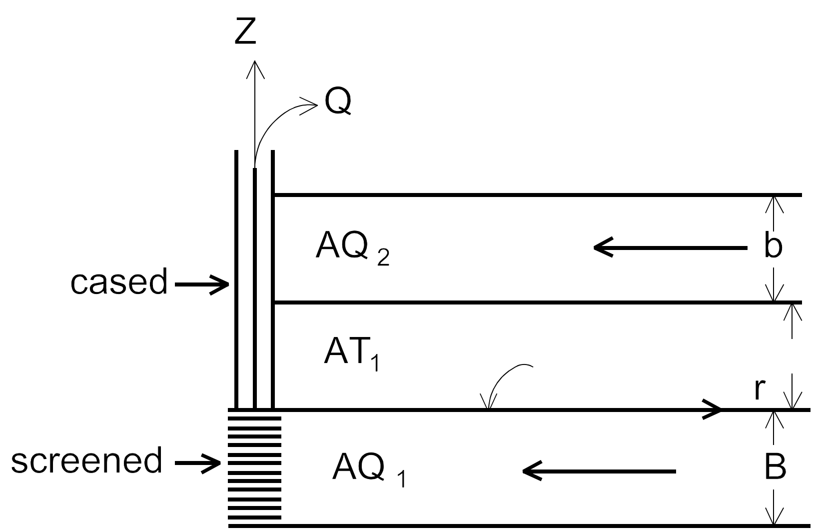
water thru AT

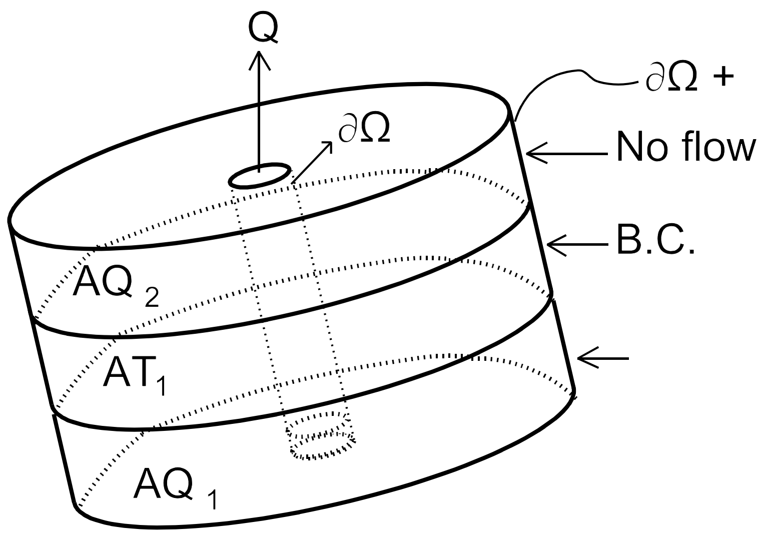
with the following initial and boundary conditions

$$s_2(r,0) = 0$$
$$s_2(\infty,t) = 0$$
$$\lim_{r\to 0} r\frac{\partial s_2}{\partial r} = 0$$

solve these P.D.E.s' for AQ_1, AT_1 and AQ_2 with the I.C. and B.C.s. Obviously the solution is complicated. It is beyond the scope of the class. If you want to be a mathematician, please see Neuman and Witherspoon (1969).

Later on, several mathematicians, Chen et al., developed solutions for flow situations that are more realistic than those examined by Neuman and Witherspoon (1969). In this case, they allowed the flow in the aquitard be fully three-dimensional but maintained a two-dimensional horizontal flow regime for both the upper and lower aquifers. Again, for details see "Exact Solution for the Problem of Cross-flow in a Bounded Two-Aquifer System with an Aquitard", by Chen, et al. (1986) *Water Resources Research* 22(8), pp. 1225-1236





$$AQ_1 \qquad \frac{\partial^2 s_1}{\partial x^2} + \frac{\partial^2 s_1}{\partial y^2} + \frac{K_Z^{'}}{T_1}\frac{\partial s_1}{\partial Z}\Big|_{Z=0} = \frac{S_1}{T_1}\frac{\partial s_1}{\partial t}$$

$$AQ_2 \qquad \frac{\partial^2 s_2}{\partial x^2} + \frac{\partial^2 s_2}{\partial y^2} - \frac{K_Z^{'}}{T_2}\frac{\partial s_1}{\partial Z}\Big|_{Z=b_1^{'}} = \frac{S_2}{T_2}\frac{\partial s_2}{\partial t}$$

$$AT_1 \qquad \frac{\partial^2 s_1^{'}}{\partial x^2} + \frac{\partial^2 s_1^{'}}{\partial y^2} - \frac{\partial^2 s_2^{'}}{\partial Z^2} = \frac{Ss_1^{'}}{K_Z^{'}}\frac{\partial s_1^{'}}{\partial t} \qquad\qquad K_x^{'} = K_y^{'} = K_z^{'}$$

B.C.

$$\frac{\partial s_1}{2n}\Big|_{\partial Q+} = \frac{\partial s_2}{2n}\Big|_{\partial Q+} \frac{\partial s_1^{'}}{2n}\Big|_{\partial Q+} = 0$$

$$\oint \partial\Omega - K_1 b_1 \frac{\partial s_1}{\partial n}\, dl = q(t)$$

I.C.

$$s_1(x, y, 0) = s_0(x, y, t)$$
$$s_2(x, y, 0) = s_0(x, y, t)$$
$$s'_1(x, y, 0) = s_0(x, y, t)$$

Interfaces

$$s'_1|_{Z=0} = s_1(x, y, t)$$
$$s'_1|_{Z=b'_1} = s_2(x, y, t)$$

$$\frac{\partial s_2}{2n} = \frac{\partial s'_1}{2n} = 0$$

Solutions → (see Chen, et al. 1986)

Results → see Figure

Conclusions

1. Significant errors in s'_1 in the AT due to vertical flow assumptions in Neuman & Witherspoon paper (1969)
2. ratio method (N & W, 1972) may not be appropriate !
3. Can't apply N & W's solutions to predict land subsidence !

Summary

Leaky Aquifers with Storativity in Aquitard $S_{S'} \neq 0$

a. Hantush, 1964, Advances in Hydroscience, edited by Chow, Vol. 1.
   1. considered S'_S in AT but const. H in AQ_2
   2. developed Asymptotic solutions

      → early time

→ late time

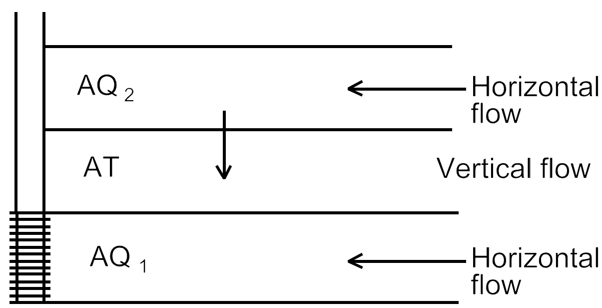
b. Neuman and Witherspoon

1. Theory of flow in a confined two aquifer system, WRR, 5(4), pp. 803-816, 1969a.
2. Applicability of current theories of flow in leaky aquifers, WRR, 5(4), pp. 817-829, 1969b.
3. Field determination of hydraulic properties of leaky multiple aquifer systems, WRR, 8(5), pp. 1284-1298, 1972.

Contributions:

1. developed a complete solution for leaky aquifer system which includes the storativity of the AT and head drawdowns in the unpumped aquifer (AQ_2)
2. Ratio Method to estimate K'_Z

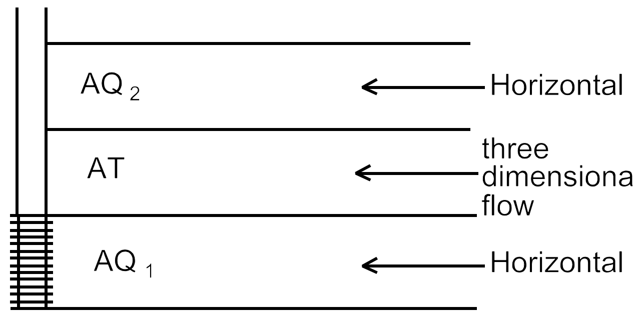
Assumptions:



Suggestions:

1. use the early time data and Theis' Solution to estimate T & S for AQ_1
2. Ratio method to estimate K'_Z

Chen, et al. (1986) "Exact Solution for the Problem of Cross-flow in a bounded Two-Aquifer system with an aquitard, WRR, 22(8), pp. 1225-1236, 1986.



1. significant errors in s'_1 in the AT due to the vertical flow assumptions in N & W's paper (1969).

2. ratio method (N & W, 1972) is not appropriate !!

CONCLUSION

Frankly, none of the above solutions are realistic enough to represent the actual flow in the field. Consequently, the estimate of the vertical hydraulic conductivity of the aquitard involves great errors. In general, the procedure for leaky aquifer is complex and the method requires installation of several additional observation wells. Furthermore, the behavior of a time-drawdown curve from a single well does not inform you the type of aquifer one encounters in the field. That is to say, a stabilizing drawdown can be attributed to many factors (for example, recharge from some recharge boundaries) other than leakage from the aquitard. The homogeneity assumption embedded in these solution further restrict the solution. Due to these reasons, we tend to stick with the Theis aquifer test analysis to estimate *T* and *S*, instead of the leaky aquifer analysis.