HWR 535 Homework #4

1. Steady-state flow in a confined aquifer with a uniform thickness, b, is described by the following equation:

$$\frac{\partial}{\partial x}\left[Kb\frac{\partial h}{\partial x}\right]=0 \qquad (1)$$

where K is the hydraulic conductivity of the aquifer, which is a constant. The boundary conditions for this flow condition are h_0 at x=0 and h_L at x = ℓ. a) Solve this equation for the head distribution along the x direction. Sketch the head as a function of x. b) Solve for h if $K=\alpha x+\beta$, where α and β are constants. Sketch the head distribution as a function of x. Explain the difference in head distribution between a) and b).

2. If the aquifer in Problem 1 is unconfined aquifer, write a proper governing partial differential equation for the problem, using the Dupuit Assumption. a) Solve the p.d.e for the same boundary conditions as in Problem 1, assuming k is constant in space. b) Solve the p.d.e for the same boundary conditions as in Problem 1, assuming $K=\alpha x+\beta$. Explain the differences in head distributions between problems 1 and 2.

3. The pressure head profile for steady-state infiltration into a soil can be described by the following solution:

$$h(z)=\frac{1}{\alpha}\ln\left[\left(e^{\alpha h_0}+\frac{q}{K_s}\right)e^{-\alpha z}-\frac{q}{K_s}\right] \qquad (2)$$

where h_0 is the prescribed boundary at $z=0$, and q is the infiltration rate, a positive quantity. The hydraulic conductivity of the soil is described by $K(h)=K_s\, exp(\alpha h)$. Graph h as a function of z for several different ratios of q/K_s (say, 1, 0.1, 0.05, 0.01, 0.001), assuming $\alpha = 0.1\ cm^{-1}$ and two different bottom boundary conditions, $h_o = 0$ and -100 cm. What happen if q/K_s becomes negative? Can you explain the result in terms of physics?

4. If the soil profile has two distinct layers with $\alpha=0.01 cm^{-1}$, $K_s = 0.01$ cm/hr for the upper layer and $\alpha=0.1 cm^{-1}$, $K_s = 0.1$ cm/hr for the bottom layer. Assume that the thickness of each layer is 1000cm and water table is at z=0. Plot h as a function of z for q=0.005 cm/hr and 0.00001 cm/hr. Carry out the same exercise if the upper layer becomes the bottom layer and the bottom layer becomes the upper layer.

5. Can you explain the difference between the mean hydraulic conductivity and the effective hydraulic conductivity?

6. If the mean and variance of lnK of an aquifer are 0 cm/hr and 1.5, what are the mean and variance of K?