Sequential aquifer tests at a well field, Montalto Uffugo Scalo, Italy

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This paper investigates our ability to characterize an aquifer using a sequential aquifer test in a well field that consists of six wells. During the test, we pumped water out from the aquifer at one well and monitored the water level changes at the rest of the wells to obtain a set of five well hydrographs. By pumping at another of the six wells, we obtained another set of five hydrographs. This procedure was repeated until each of the six wells was pumped. We then analyzed the six sets of hydrographs using the classical drawdown-time, the drawdown-distance, and the drawdown-distance/time methods.

Results of the analysis confirm recent findings that the transmissivity estimates vary significantly at early time and stabilize at late time. At late time, the estimated values from all hydrographs are similar overall but vary slightly according to the locations of the pumping and observation wells. In contrast, storage coefficient estimates stabilized rapidly to distinct values associated with the well locations. We subsequently used a hydraulic tomography approach to include all hydrographs from the sequential aquifer test to estimate the spatially varying transmissivity and storage coefficient fields. The estimated fields appear to be realistic: They reflect the geologic setting and the behaviors of the well hydrographs, although more definitive confirmation is needed.


1. Introduction

Knowledge of detailed spatial distributions of hydraulic properties is imperative to improve our ability to predict water and solute movement in the subsurface at high resolutions [e.g., Yeh, 1992, 1998]. Over the past few decades, the characterization of aquifers has relied on traditional aquifer test methods (i.e., cross-hole tests: pumping at one well and observing the response at another well, and the use of Theis' or Jacob's analysis [Theis, 1935; Cooper and Jacob, 1946]) or slug tests. Traditional aquifer tests are thought to yield averaged hydraulic properties over a large volume of geologic media [e.g., Butler and Liu, 1993]. In actuality, the classical analysis for aquifer tests yields spurious average transmissivity values that are difficult to interpret. On the other hand, these traditional tests and analyses yield storage coefficient estimates that reflect the local geology between the pumping and the observation well [Wu et al., 2005, Butler [1997], Beckie and Harvey [2002], and others have mentioned that values for storage estimated by means of single well slug tests have to be treated with care.

In spite of these controversies, high-density measurements of hydraulic properties of an aquifer using traditional aquifer test methods over a large basin are deemed prohibitive and impractical. On the other hand, measurements of spatially distributed water level responses (i.e., well hydrographs) to either natural or artificial excitations of an aquifer generally are less costly and relatively abundant. Making use of these well hydrographs to identify the spatial distribution of hydraulic properties of an aquifer (i.e., inverse modeling of distributed parameter fields or “inverse modeling” for short) is therefore rational. Nevertheless, without sufficient data to meet necessary and sufficient conditions of the inverse problem, the problem can be ill posed, and the solution to the problem (i.e., the estimated hydraulic property fields) will be nonunique.

To improve the uniqueness of the inverse solution and reduce uncertainties in the identified hydraulic property field, several researchers have investigated the use of data corresponding to different flow situations [Scarascia and Ponzini, 1972; Giudici et al., 1995; Snodgrass and Kitanidis, 1998]. More recently, a relatively new method has been developed, called hydraulic tomography [Gottlieb and Dietrich, 1995; Renshaw, 1996; Vasco et al., 2000; Yeh and Liu, 2000; Liu et al., 2002; Bohling et al., 2002; McDermott et al., 2003; Brauchler et al., 2003], which adopts the concept of the computerized axial tomography (CAT) scan in medical sciences and geophysical tomographic surveys. This new method appears to be a viable technology for characterizing detailed spatial distributions of hydraulic properties over a large volume of geologic media, without resorting to a large number of wells.

Hydraulic tomography is, in the most simplified terms, a series of cross-well interference tests (i.e., sequential aquifer tests) combined with a simultaneous interpretation of the tests to estimate the spatial distribution of hydraulic
properties of an aquifer. Specifically, an aquifer is stressed by pumping water from or injecting water into a well (pressure excitation or stress), and the aquifer response is monitored at other wells. A set of stress/response data yields an independent set of equations. Sequentially switching the pumping or injection location, without installing additional wells, results in a large number of aquifer responses caused by stresses at different locations and, in turn, a large number of independent sets of equations. This large number of sets of equations makes the inverse problem better posed (constrained), and the estimate will be closer to reality than the traditional inverse approaches. In this context, hydraulic tomography analysis is different from the analysis of traditional cross-hole tests that use solutions that assume aquifer homogeneity.

Interpreting data from hydraulic tomography presents a challenge, however. The abundance of data generated during tomography often leads to information overload and causes substantial computational burdens and numerical instabilities [Hughson and Yeh, 2000]. Moreover, results of hydraulic tomographic analysis can be nonunique. In order to overcome these difficulties, Yeh and Liu [2000] developed a sequential successive linear estimator (SSLE). This estimator eases the computational burdens by sequentially including information obtained from different aquifer tests; it reduces the nonuniqueness problem by providing the best unbiased conditional mean estimate. That is, it conceptually organizes hydraulic parameter fields as spatial stochastic processes and seeks their mean distributions conditioned on the information obtained from hydraulic tomography as well as parameter values obtained from slug tests or core samples and our prior knowledge of spatial statistical structure of the heterogeneity. Using sandbox experiments, Liu et al. [2002] demonstrated that the combination of hydraulic tomography and SSLE is a propitious, cost-effective technique for delineating hydraulic conductivity heterogeneity using a limited number of invasive observations.

The work by Yeh and Liu [2000] is limited to steady state flow conditions, which may occur only under special field conditions. Because of this restriction, their method ignores the transient head data that occur before flow reaches steady state conditions. Transient head data, although influenced by both hydraulic conductivity and specific storage, are less likely to be affected by uncertainty in boundary conditions. As a result, Zhu and Yeh [2005] developed an SSLE algorithm for transient hydraulic tomography to estimate three-dimensional distributions of both hydraulic conductivity and specific storage fields. Recently, Liu et al. [2007] validated the transient hydraulic tomography concept and the SSLE algorithm using sandbox experiments. In particular, they showed that using the estimated hydraulic conductivity and specific storage fields, they can predict reasonably well the temporal and spatial distributions of drawdown induced by an independent pumping test. They thereby concluded that transient hydraulic tomography is a promising tool for detailed aquifer characterization. But transient hydraulic tomography has not yet been extensively applied to field problems. Therefore the utility of hydraulic tomography for field situations remains to be tested and assessed.

In this study, we conducted a sequential aquifer test using six wells in a confined aquifer at a field site, Montalto Uffugo Scalo, Italy. The well hydrographs collected during the test were analyzed with a traditional approach (drawdown-time analysis), as well as a distance-drawdown and a drawdown-distance-time approach to define the effective transmissivity and the storage coefficient. The objective of these analyses is to verify recent findings by Wu et al. [2005] regarding the traditional aquifer test and analysis. Specifically, we want to determine if, in response to a pumping well, the Theis analysis of a hydrograph at an observation well will yield an averaged transmissivity value that varies with well locations. We also want to confirm that storage coefficient estimates reflect local properties between the pumping and observation wells.

Following the above analyses, we applied to the data sets Zhu and Yeh’s [2005] SSLE for transient hydraulic tomography. This application aims to identify spatially varying transmissivity and storage coefficient fields rather than the equivalent homogeneous properties as in the traditional analyses. The validity of the estimated T and S fields was qualitatively checked for their consistency with geologic information and with an analysis of the derivative of the drawdown with respect to the log of time.

2. Description of the Field Site

The study area is a groundwater test site of the Soil Conservation Department of the University of Calabria, near the town of Montalto Uffugo, about 300 km south of Naples, Italy. The test site is located in an alluvial deposit at the confluence of the Settimo River in the south, the Mavigliano River in the north, and the Crati River in the east. The deposit consists mainly of unconsolidated and highly permeable alluvial sands and conglomerates. The well field of the test site encompasses an area about 2100 m² (35 m × 60 m), and the subsurface geology of the site has been classified in to four geological units (Figure 1). The top unit
(formation A) is composed of heterogeneous gravels embedded in a silty sand matrix. This formation extends from the ground surface to a depth of about 7 m. Underlying the top formation is a shale layer (formation B) at approximately 7- to 11-m depth. The third formation (formation C), at approximately 11- to 55-m depth, is the main aquifer of our interest. It is composed mainly of silty sand. Underlying formation C is a low-permeability shale stratum (formation D). A shallow perched aquifer is sometimes present in formation A during part of the year. This was the case at the time of our experiment. The main aquifer is weakly confined: The piezometric surface was approximately 4 m above the top of formation B.

The groundwater monitoring facility, established in June 1993, consists of five monitoring stations: A central monitoring station is surrounded by four additional monitoring stations. Each station includes a borehole reaching a depth of 10 m (i.e., reaching the shallow perched groundwater) and a second borehole reaching a depth of 40 m in the aquifer of interest. All the boreholes have a metal casing; the shallow boreholes are screened over an interval of 2 m, whereas the deep boreholes have a screened portion of 17 m. The boreholes are numbered P1 to P10 (Figure 1).

A new well was drilled in October 1997 to a depth of 57 m to penetrate the clay bottom of the confined aquifer. This additional borehole, P11, is located 19 m from the central well (P5), and it reaches the impermeable layer at the bottom of the main aquifer. All the wells have a diameter of 20 cm. It should be emphasized here that the description in the previous paragraph and the sketch of the geology of the field site (Figure 1) is only a conceptual model that is built upon drilling logs from these limited number of wells, and the true lateral distribution of these strata is unknown.

3. Sequential Aquifer Tests

Several aquifer tests were conducted in the Montalto well field from the early 1990s until today [Fallico et al., 2002; Rizzo et al., 2004; Titov et al., 2005]. In this paper, we discuss a sequential aquifer test at the Montalto well field, which began in October 2001 and terminated in February 2002. During the sequential aquifer test, water was pumped from one of the six wells, and drawdown-time curves were measured at the remaining five wells. Afterward, the pump was shut off to allow the aquifer to recover. The pump was then moved to another well, and the same procedure was repeated. Six aquifer tests were conducted, yielding 30 drawdown-time data sets; no water level measurements were taken at the pumping well. For all the aquifer tests, the pumping rate was restricted to about 1 L per second to ensure that the piezometric surface remained within the 4 m above the top of formation B so that the aquifer remained confined. The pumping rate was kept constant during all the aquifer tests.

Figure 2 shows the semilog plots of drawdown-time data of five observation wells while pumping was conducted at one of the six wells during the six aquifer tests. As illustrated in this figure, all the drawdown-time curves do not exhibit straight-line behaviors at large time, as one might expect, based on depictions in any textbook for homogeneous aquifers with infinite lateral extents. Instead, they exhibit continuous change in the slope. While most of the drawdown data show a continuous increase in slope as a function of time, some started with a steep slope and then changed to a less steep curve at later times. At large times, all of the plots show an increase in slope, suggesting existence of some higher-permeability regions within or adjacent to lower-permeability regions. Because of the heterogeneity and the distances between the wells, the arrival times of drawdown (the intercepts of the tangent of the drawdown-time curves) are different for each well during different aquifer tests. In particular, notice that well P5 is located near the center of the well field: The distances between this well and other wells P1, P3, P7, and P9 are almost identical. However, when well P5 was pumped, the four other wells responded differently (see Figure 2), evidence of the heterogeneous nature of the aquifer (more precisely, deviation from the Theis solution). Likewise, in contrast to some of the findings in theoretical analysis of drawdown in aquifers of statistically stationary random parameters [e.g., Indelman, 2003; Meier et al., 1998; Sánchez-Vila et al., 1999], none of these drawdown-time data has an “identical” slope at late time.

We also note that during all pumping tests (except the test with pumping at well P9), the water level observed at well P9 increased initially and then decreased as expected. This was attributed to poor backfill between the borehole and well casing, which caused leakage from the upper perched aquifer into the well. As a result, only the drawdown portions of the hydrographs at well P9 are shown in these figures.

4. Methods of Analysis

The drawdown-time data for the six aquifer tests were analyzed by several methods which are based on either homogeneous or heterogeneous assumptions. First, assuming that the aquifer is homogeneous, we analyzed the well hydrographs of the aquifer tests using (1) a classical drawdown-time analysis, (2) a drawdown-distance analysis, and (3) a drawdown-distance/time analysis. Afterward, without invoking the aquifer homogeneity assumption, the data set was analyzed by the SSLE algorithm developed for hydraulic tomography analysis to obtain spatially distributed T and S estimates of the field site. These methods are explained below. Notice that all these methods employ a depth-average concept that ignores vertical flow as well as vertical hydrologic heterogeneity, including variations in thickness. Similarly, we assume that all the wells fully penetrate the aquifer. Validity of this assumption is unknown since the detailed geology of the site itself is unknown.

4.1. A Classical Drawdown-Time Analysis

In this section, we estimate T and S values of the aquifer using a traditional method of analysis for aquifer tests. That is, we employ a nonlinear least squares approach to minimize the following objective function:

$$\chi^2 = \sum_{j=1}^{t_{\text{max}}} \frac{(w(r, t_j) - \hat{w}(r, t_j))^2}{\sigma(r, t_j)} = \text{min}$$

where $w(r, t_j)$ and $\hat{w}(r, t_j)$ are the observed and theoretical drawdown at time $t_j$ and distance $r$; $t_{\text{max}}$ is the maximum observation time of the hydrograph at $r$; $\sigma(r, t_j)$ is the weight, representing measurement errors. In the following
analysis, the weight was set to 1. The theoretical drawdown is calculated using the Theis solution [Theis, 1935], which assumes horizontal flow, aquifer homogeneity, uniform thickness, and infinite lateral extents of the aquifer. Because the screen intervals are the same and at the same levels for each well and more importantly the uncertain geology, we assume that the effects of partial penetrating well is minimal. The Theis solution is:

\[ w(r, t) = \frac{Q}{4\pi T} W(u) \]  

with \( u = \frac{r^2 S}{4T} \), where \( Q \) is the given pumping rate. Equations (1) and (2) were used to estimate \( T \) and \( S \) values from each observed hydrograph at five observation wells induced by each of the six pumping events.

4.2. A Drawdown-Distance Analysis

[17] The drawdown-distance analysis aims to estimate effective \( T \) and \( S \) for each aquifer test during the sequential aquifer tests. In other words, we seek effective \( T \) and \( S \) values that can simultaneously minimize the difference between the observed and theoretical drawdown at all the

Figure 2. Measured drawdown at five observation wells as a function of time during each of the six aquifer tests.
observation wells at a given time \( t \) during an aquifer test. The objective function to be minimized is

\[
\chi^2 = \sum_{i=1}^{n} \left( \frac{w(r_i, t) - \hat{w}(r_i, t)}{\sigma(r_i, t)} \right)^2 = \min
\]  

(3)

where \( w(r_i, t) \) is calculated by equation (2); \( r_i \) denotes the distance between the pumping and observation wells \( i; \) \( n \) is the total number of observation wells. This method is a theoretically consistent approach to define the effective parameters for an equivalent homogeneous formation as demonstrated by Wu et al. [2005] for radial flow in aquifers.

4.3. A Drawdown-Time-Distance Analysis

[18] Another approach used to estimate the effective \( T \) and \( S \) is a simultaneous regression of all the drawdown induced by pumping at a well and observed at all other wells, from time zero to any given time. Specifically, this approach is based on the minimization of the following objective function:

\[
\chi^2 = \sum_{j=1}^{J} \sum_{t=1}^{T} \left( \frac{w(r_j, t) - \hat{w}(r_j, t)}{\sigma(r_j, t)} \right)^2 = \min
\]  

(4)

where the well index \( i = 1, \ldots, n \) which represents the total number of observation wells; \( j \) denotes the time index of the drawdown data; \( \hat{w}(r_j, t) \) are the observed drawdowns during a given aquifer test in the observation well \( i \) at time \( t \); \( w(r_j, t) \) are the drawdowns calculated from equation (2).

4.4. Hydraulic Tomography Analysis

[19] Instead of assuming a homogeneous aquifer, we synthesized the results of all aquifer tests using the hydraulic tomography approach to depict the spatial distribution of \( T \) and \( S \) values. That is, we first estimated a \( T \) and \( S \) field using a set of drawdown-time data at five observation wells during pumping at a selected well. The estimated fields were subsequently improved by incorporating another set of drawdown-time data induced by pumping at a different pumping well. These estimates were successively modified until all six sets of the aquifer test data were utilized. To conduct this hydraulic tomography analysis, we used the stochastic estimator (i.e., SSLE) by Zhu and Yeh [2005] which is briefly described below.

[20] To characterize the heterogeneity of geologic formations, the SSLE algorithm considers the natural logs of transmissivity and the storage coefficient as spatial stochastic processes. Therefore \( \tilde{T}(x) = \exp[\tilde{F}(x) + f(x)] \) and \( \tilde{S}(x) = \exp[\tilde{G}(x) + g(x)] \), where \( x \) is the location vector, \( F \) and \( G \) are means of the log of \( T \) and \( S \) values, and \( f \) and \( g \) denote the perturbations. The transient hydraulic head response to an aquifer test is represented by \( \tilde{H}(x) = \tilde{H}(x) + \tilde{h}(x) \), where \( \tilde{H} \) is the mean and \( \tilde{h} \) is the perturbation. Substituting these stochastic variables into a two-dimensional depth-averaged equation, taking the conditional expectation, and conditioning with some observations of head and parameters generate the mean flow equation as

\[
\nabla \cdot [\bar{T}(x) \nabla \bar{H}(x)] + \mathbf{Q}(x_p) = \bar{S}(x) \frac{\partial \bar{H}(x)}{\partial t}
\]  

(5)

where \( \bar{T}(x) \), \( \bar{H}(x) \), and \( \bar{S}(x) \) are the conditional effective transmissivity, the hydraulic head, and storage coefficient, respectively [Yeh et al., 1996]. According to equation (5), the SSLE method then seeks the conditional effective fields of transmissivity and the storage coefficient, conditioned on the information from transient hydraulic tomography surveys and some estimates of \( T \) and \( S \), if any, obtained from slug tests or core samples.

[21] Specifically, the estimation procedure starts with a weighted linear combination of direct measurements of the hydraulic properties, if any, and drawdown-time data at different observation locations due to the first aquifer test to obtain the first estimate of the parameters. The weights are calculated based on statistical moments (namely, means and covariances) of parameters, the covariances of heads in space and time, and the cross-covariances between heads and parameters. After the estimate, the covariances are updated to reflect the effects of conditioning, and they are called residual (or conditional) covariances, representing the uncertainty associated with the estimate. This estimation procedure is identical to cokriging. Afterward, the first estimate is used in the mean flow equation (5) to calculate the new heads at observation locations and sampling times (i.e., forward simulation). Differences between the observed and simulated heads are determined subsequently. A weighted linear combination of these differences is then used to update the previous estimates. During the update, the weights are evaluated using residual (conditioned) covariances and cross-covariances which are updated at each iteration. Iterations between the forward simulation and estimation continue until the improvement in the estimates diminishes to a prescribed value. The above procedures are repeated for each aquifer test until the data from all the tests are exhausted. Detailed algorithm and procedures are given in the work of Zhu and Yeh [2005].

[22] To apply the SSLE to the field site, a square area of 100 m \( \times \) 100 m was selected and discretized into 50 \( \times \) 50 elements, with an element size of 2.0 m \( \times \) 2.0 m. The dimensions of the domain take into account the radius of influence determined during previous aquifer tests [Troisi and Straface, 1996; Troisi et al., 2000]. Application of the SSLE model requires statistics of the spatial variability of the aquifer hydraulic properties, i.e., the mean, variance, correlation scales, and theoretical model of covariance. A mean value of \( 1.69 \times 10^{-4} \) m/s for \( T \) and 0.0072 for \( S \) were used based on the results from the classical drawdown-time-distance analysis illustrated in the previous section. On the basis of the findings by Zhu and Yeh [2005], three drawdown measurements were selected to estimate parameters. They covered the early, intermediate, and late times of each hydrograph during each aquifer test. The measurement times ranged from 1800 to 108,000 s. A total of 87 drawdown measurements were used.

5. Results and Discussions

[23] Tables 1 and 2 list the estimated \( T \) and \( S \) values, respectively, using traditional drawdown-time analysis based on the 30 cross-well hydrographs. The top row indicates the observation well, and the left column lists the pumping location. No estimate was obtained for the pumping well. Means and standard deviations of the estimates based on the hydrographs observed at the same well during tests at different locations are tabulated at the bottom two rows of the tables. Similarly, means and standard
Table 1. Estimated $T$ Values (in $10^{-4}$ m$^2$/s) Using the Classical Drawdown-Time Analysis (W-T), the Drawdown-Distance Analysis at the Final Time (W-R), and Drawdown-Distance-Time Analysis at the Final Time (W-T-R)

<table>
<thead>
<tr>
<th>Pumping Wells</th>
<th>OB1</th>
<th>OB3</th>
<th>OB5</th>
<th>OB7</th>
<th>OB9</th>
<th>OB11</th>
<th>Mean</th>
<th>Stdev</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>–</td>
<td>–</td>
<td>1.250</td>
<td>1.275</td>
<td>1.417</td>
<td>1.317</td>
<td>1.314</td>
<td>1.315</td>
<td>0.064</td>
</tr>
<tr>
<td>3</td>
<td>1.923</td>
<td>–</td>
<td>1.943</td>
<td>1.976</td>
<td>1.875</td>
<td>1.841</td>
<td>1.912</td>
<td>0.054</td>
<td>2.240</td>
</tr>
<tr>
<td>5</td>
<td>2.809</td>
<td>2.465</td>
<td>–</td>
<td>3.007</td>
<td>2.708</td>
<td>2.799</td>
<td>2.758</td>
<td>0.197</td>
<td>4.770</td>
</tr>
<tr>
<td>7</td>
<td>1.441</td>
<td>1.273</td>
<td>1.425</td>
<td>–</td>
<td>1.336</td>
<td>2.047</td>
<td>1.504</td>
<td>0.311</td>
<td>1.650</td>
</tr>
<tr>
<td>9</td>
<td>1.320</td>
<td>1.410</td>
<td>1.401</td>
<td>1.310</td>
<td>–</td>
<td>1.431</td>
<td>1.374</td>
<td>0.055</td>
<td>2.337</td>
</tr>
<tr>
<td>11</td>
<td>1.422</td>
<td>1.484</td>
<td>1.482</td>
<td>n.a.</td>
<td>1.306</td>
<td>–</td>
<td>1.424</td>
<td>0.083</td>
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</tr>
<tr>
<td>Mean</td>
<td>1.783</td>
<td>1.576</td>
<td>1.505</td>
<td>1.928</td>
<td>1.708</td>
<td>1.886</td>
<td>1.697</td>
<td>1.306</td>
<td>0.563</td>
</tr>
<tr>
<td>Stdev</td>
<td>0.619</td>
<td>0.506</td>
<td>0.256</td>
<td>0.777</td>
<td>0.608</td>
<td>0.573</td>
<td>1.135</td>
<td>1.001</td>
<td></td>
</tr>
</tbody>
</table>

Table 2. Estimated $S$ Values (in $10^{-3}$) Using Classical Drawdown-Time Analysis (W-T), the Drawdown-Distance Analysis at the Final Time (W-R), and Drawdown-Distance-Time Analysis at the Final Time (W-T-R)

<table>
<thead>
<tr>
<th>Pumping Wells</th>
<th>OB1</th>
<th>OB3</th>
<th>OB5</th>
<th>OB7</th>
<th>OB9</th>
<th>OB11</th>
<th>Mean</th>
<th>Stdev</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>2.165</td>
<td>–</td>
<td>2.832</td>
<td>0.654</td>
<td>11.404</td>
<td>2.027</td>
<td>3.816</td>
<td>4.315</td>
<td>1.009</td>
</tr>
<tr>
<td>5</td>
<td>8.965</td>
<td>19.943</td>
<td>–</td>
<td>11.977</td>
<td>44.459</td>
<td>5.403</td>
<td>18.167</td>
<td>15.693</td>
<td>5.933</td>
</tr>
<tr>
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<td>0.965</td>
<td>2.592</td>
<td>1.545</td>
<td>n.a.</td>
<td>19.476</td>
<td>–</td>
<td>6.144</td>
<td>8.913</td>
<td>1.001</td>
</tr>
</tbody>
</table>

deviations of the estimates from the hydrographs observed at five wells during a pumping test at each pumping location are tabulated in the middle columns of the tables. According to these tables, the estimated $T$ and $S$ values vary slightly with the pumping and the observation locations. This result seems consistent with the conclusion of Wu et al. [2005] that the $T$ estimate based on the traditional analysis represents some average over the cone of depression, and the average is influenced by geologic heterogeneity near the pumping and observation wells. The tabulated means of the $T$ estimates, based on observed hydrographs induced by pumping at each different pumping location, shows that the average values are high when wells P3, P5, and P7 were pumped. On the other hand, variation in the mean of the $T$ estimates based on the hydrographs observed at wells P3, P5, and P7 due to pumping at different wells is small. On the basis of the finding by Wu et al. [2005] that the $T$ estimate likely is influenced by the geology near the wells, these results perhaps are indicative of existence of high permeability zones near wells, P3, P5, and P7.

[24] According to Wu et al. [2005], the $S$ estimate based on the traditional analysis reflects the $S$ properties between the pumping and observation wells. Results in Table 2 appear to corroborate their conclusion. For example, the $S$ estimate using the hydrograph of well P5 caused by pumping at well P1 is high, and likewise, the estimate based on observed hydrographs at well P1 caused by pumping at well P5 is large. Similarly, the estimate using the hydrograph of well P11 induced by pumping at well P1 is low, so is the estimate based on well P1 hydrograph due to pumping at well P11. This consistency exists for some other pairs of wells (for example, pumping well P5 and observation well P9, pumping well P9 and observation well P5, pumping well P9 and observation well P11, and pumping well P11 and observation well P9), and it is illustrated in Figure 3. In addition, the mean values of $S$ estimates derived from observation wells are high when wells P1, P5, and P9 were pumped.

[25] The variations in the means and variances of the $T$ estimates are generally smaller than those of the $S$ estimates. Overall, the traditional drawdown-time analysis that rests upon aquifer homogeneity assumption yields estimates of aquifer properties that are difficult to interpret.

[26] Now we examine the estimates of $T$ and $S$ using the drawdown at different times at each well, and they are plotted in Figures 4 and 5, respectively, indicated by a solid line marked with the well number. According to these figures, the estimates of $T$ and $S$ change with time: $T$ estimates based on different observation wells tend to converge to some value as the pumping duration increases, but the final estimates are not the same, although differences between them are small. $S$ estimates stabilized rather quickly and reach their own distinct values that are significantly different from each other comparing to the variation in $T$ estimates. Furthermore, the behaviors of the estimates are different for each different pumping location, based on the hydrographs induced by pumping at a given well.

[27] After estimating $T$ and $S$ using the traditional drawdown-time analysis, we analyzed snapshots of cones of depression to derive effective aquifer properties. The snapshots were based on the drawdown recorded at five observation wells at a given time, and the effective properties were derived using the drawdown-distance analysis. Figure 4 shows the estimates of the effective $T$ at different time for the six aquifer tests. The effective $S$ estimates by
the drawdown-distance analysis are not plotted in Figure 5 because their great variability is out of the range. On the basis of Figure 4, the effective $T$ estimates generally decrease with time, although they have some oscillations, likely because only five drawdowns (an insufficient number of observations) for a given pumping location were used at a given time. The estimates of the effective $T$ and $S$ at the largest time are tabulated in Tables 1 and 2, respectively, for each aquifer test. The tabulated values indicate that the effective $T$ and $S$ are the greatest when well P5 was pumped. The variation of the estimates of the aquifer tests using different pumping locations suggests the nonexistence of the effective parameters due to the fact that only five hydrographs were used [see Wu et al., 2005] and possibly the fact that the effective parameter may vary with the geology around the pumping well.

[28] Estimates of effective $T$ and $S$ as a function of time using the drawdown-time-distance analysis are shown as solid lines with triangles in Figures 4 and 5, respectively. Unlike the previous distance-drawdown analysis, this approach minimizes the difference between the simulated hydrograph using the estimated $T$ and $S$ and five observed hydrographs from a starting time to a given observation time. As a result, each pair of estimates provides the best fit of the five observed hydrographs not only at the given time but also those at previous time. Accordingly, the estimates plot smoothly in these figures. Nevertheless, the estimates are different for pumping at different wells. The $T$ and $S$ estimates at the final time are tabulated in Tables 1 and 2 again. As indicated in the tables, the effective $T$ and $S$ estimates are the highest for the aquifer test at well P5. This result is somewhat consistent with the results shown for drawdown-distance analysis.

[29] Overall, the results based on the traditional analysis, drawdown-distance analysis, and drawdown-distance-time analysis appear to agree with findings by Wu et al. [2005] and disagree with the findings by Meier et al. [1998] and Sánchez-Vila et al. [1999]. In other words, it is difficult to derive the true effective aquifer properties from a hydrograph observed at a single well. To overcome this difficulty, hydrographs from densely distributed monitoring wells must be used in conjunction with the spatial moment or the distance-drawdown approach [see Wu et al., 2005]. This also raises a salient question about the validity of applications of any stochastic approach to estimation of spatial variability of the aquifer properties based on few observation hydrographs induced by one aquifer test [e.g., Neuman et al., 2004]. Theories for estimation of spatial variability that take advantages of sequential aquifer tests would yield more reliable estimates.

[30] Finally, estimated $T$ and $S$ distributions based on hydraulic tomography analysis are illustrated in Figures 6 and 7, respectively. The computational time for the analysis was approximately 1-hour CPU time using an 8-node PC cluster. In Figure 6, the well field is located in a large isolated permeable island that is surrounded by low transmissivity materials. There are some highly permeable patches within this island, and this perhaps explains the flattening behaviors of some of the drawdown-time curves. Since the estimate is for transmissivity, a depth-averaged
aquifer property, the low transmissivity zone around the island is perhaps an indication of the pinch-out of the aquifer in the lateral directions. The estimated $T$ field and explanation appear to be consistent in view of the fact that this well field is located in an alluvial deposit at the junction of three rivers. However, we cannot offer any explanation for the distribution of the estimated $S$ field: The storage coefficient does not necessarily correlate with the transmissivity. Since drawdown is strongly influenced by the $S$ field between the pumping and the observation wells according to Wu et al. [2005], we postulate that resolution of the estimated $S$ field is likely limited to the area surrounded by

Figure 4. Estimated $T$ values using drawdown-distance analysis at different times (triangles) using hydrographs at different wells (the number embedded in each curve refers to the observation well number, i.e., 1 stands for well P1, and so on. B is used to denote well P11).
the well cluster. The geometric means of estimated $T$ and $S$ fields are $3.26 \times 10^{-5}$ m$^2$/s and 0.002, respectively. The variances of the estimated $\ln T$ and $\ln S$ for the field site are 5.145 and 3.569, respectively, echoing the suggestion by Fogg [2004] that past investigations have underestimated the variability of $T$ fields.

To further substantiate the estimated $T$ field from the hydraulic tomography analysis, we analyzed the drawdown rate, the derivative of the drawdown with respect to the

**Figure 5.** Estimated $S$ values using drawdown-distance analysis at different times (triangles) using hydrographs at different wells (the number embedded in each curve refers to the observation well number, i.e., 1 stands for well P1, and so on. B is used to denote well P11).
Figure 6. Estimated transmissivity distribution using the hydraulic tomography approach with possible locations of change in transmissivity calculated by means of the derivative of the drawdown with respect to the log of time and the Oliver relation for the aquifer test at well P7.

Figure 7. Estimated storage coefficient distribution using the hydraulic tomography approach.
ln(t): d\delta/d\ln t = t d\delta/dt, rather than the actual drawdown, because the drawdown rate is highly sensitive to the variability in the transmissivity field [e.g., Copty and Findikakis, 2004]. That is, the drawdown rate [the derivative of the drawdown with respect to the ln(t)] is a very important quantity that bears the signature of the heterogeneity and the boundary effects. In particular, if the drawdown-ln(t) curve is concave, its first derivative is positive and is negative if the drawdown curve is convex. Where this curve goes from a concave to a convex shape, we have an inflection point, corresponding to a minimum or maximum of the derivative curve. When heterogeneity is encountered by the drawdown front, an inflection point appears in the drawdown-ln(t) curve and, in turn, a relative maximum or minimum in the drawdown rate curve. Therefore by analyzing the drawdown rate curve, we may estimate the location of the heterogeneity or the boundary. Oliver [1990] provided a formula for this estimate, which is valid for a small permeability variation from an average value.

\[ r = 0.015 \left[ \frac{E_t}{\phi \mu c_i} \right]^{1/2} \]  

where \( r \) is the radial distance between the pumping well and the boundary or heterogeneity (m), \( \bar{k} \) is the average intrinsic permeability (\( \mu \)m), \( t \) is the time (hr), \( \phi \) is the porosity (\( \mu \)), \( \mu \) is the dynamic viscosity (Pa s), and \( c_i \) is the total compressibility (Pa\(^{-1}\)).

[32] Equation (6) was applied to the sequential aquifer test data to assess the transmissivity estimates. To illustrate the procedure, the analysis of the derivative of the drawdowns measured during the aquifer test at the well 7 with respect to the ln(t) is discussed. This particular aquifer test was carried out on 15 January 2002. The duration of this aquifer test was approximately 43 hours, with a constant pumping rate of 0.9 L/s. In Figure 8, we show the behavior of the derivative of the drawdown with respect to ln(t) for every observation well. In this figure, the curves for drawdown rate show two maxima and two minima. The times at which these inflection points occurred (i.e., the time at which transmissivity variations are encountered by the drawdown) are, for all observation wells, approximately 4, 11, 24, and 38 hours from the beginning of the well 7 aquifer test. By means of equation (6), we calculated the radial distance between the pumping well and the possible locations of change in transmissivity. The distances are 21.1, 33.9, 48.9, and 63.5 m. Next, we plotted a circle corresponding to each of these radial distances on the map of transmissivity estimated by SSLE (see Figure 6). According to this figure, the locations of different transmissivity zones depicted by SSLE are strikingly consistent with those estimated by means of the drawdown rate and Oliver’s [1990] relation. Notice that the Oliver radii formula is a weak approximation of radial flow in the depth-averaged aquifers with concentric heterogeneity. Oliver’s approach presented likely locations of change in transmissivity, but the SSLE mapped a likely distribution of transmissivity simultaneously using the five aquifer test data sets.

[33] The same procedure was applied to the other four aquifer tests. The estimated distances between the pumping well and the heterogeneities are tabulated in Table 3. Again,
the results are consistent with the transmissivity map derived from the SSLE.

6. Conclusions

[34] All the well hydrographs observed during the sequential aquifer tests exhibit effects of heterogeneity. Hydrographs at the same observation well during each pump test at a different distance from the observation well behaved differently excluding effects of distance. Likewise, hydrographs observed at wells at the same distance from a pumping well also acted differently. These hydrographs deviate from the responses predicted by the Theis solution, which assumes aquifer homogeneity. In spite of the deviations, applications of the solution (drawdown-time analysis) to these hydrographs yielded different, temporally evolving \( T \) and \( S \) values for each observation well during pumping at a given well. The \( T \) estimates stabilize at large times and converge to slightly different values for different observation wells. These estimates also vary with the pumping location. Estimated \( S \) values diverge but stabilize to a distinct value for each test and observation well. These results support the findings by Wu et al. [2005]. Variation in estimates was also observed when drawdown-distance and drawdown-time-distance approaches were employed to estimate effective \( T \) and \( S \). This variation may be attributed to the limited number of observation wells and perhaps to the fact that true effective properties also depend on the location of the pumping well. We thereby question the validity of applications of the traditional aquifer test analysis to cross-hole aquifer tests in heterogeneous aquifers. The traditional method nonetheless is important in context of aquifer characterization. For example, these methods can be used to obtain first estimates of aquifer properties quickly and inexpensively. It is possible to evaluate pumping tests in the field and, based on these results, to plan the further test procedure. In addition, the \( T \) and \( S \) values estimated with traditional evaluation methods can be used as starting parameters for the tomographical inversion. By the same token, we also question the validity of applications of stochastic approaches based on one or two hydrographs induced by one aquifer test to estimate spatial statistics of aquifer properties. New approaches that take advantage of HT data to estimate the spatial statistics of aquifer would be more meaningful.

[35] Last, this study demonstrates that a sequential aquifer test unequivocally produces more information than a classical aquifer test using the same well field. The sequential aquifer test collected data intelligently: Each drawdown history observed at different wells represents a view of the heterogeneity at different angles and perspectives. Using these observed hydrographs, our study shows that the hydraulic tomography analysis algorithm [Zhu and Yeh, 2005] yields reasonable estimates of spatially varying \( T \) and \( S \) fields. These estimated fields reflect the possible pattern of boundaries or changes in properties as exhibited in the hydrograph and are considered to be consistent with the geologic setting of the site. In spite of these consistencies, more definitive confirmations are needed. Overall, hydraulic tomography reveals a heterogeneity pattern using only five wells, and this is beyond the ability of traditional approaches. Therefore we believe that applications of the sequential aquifer test (or hydraulic tomography) can lead to improvement in characterization of the subsurface and, in turn, perhaps better groundwater management.

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References


Cooper, H. H., Jr., and C. E. Jacob (1946), A generalized graphical method for evaluating formation constants and summarizing well-field history, J. Geol., 54, 256 – 253.


Illman, W. A., X. Liu, and A. Craig (2006), Steady state hydraulic tomography in a laboratory aquifer with deterministic heterogeneity.
Multi-method and multiscale validation of hydraulic conductivity tomograms, submitted manuscript.


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