Steady-state discharge into tunnels in formations with random variability and depth–decaying trend of hydraulic conductivity

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ABSTRACT

Multi-scale heterogeneity of geological formations is a rule, which consists of random (local-scale) and systematic (large-scale) variability of hydraulic conductivity. The random variability and depth–decaying trend, a systematic variability, have different effects on subsurface flow, thus on groundwater discharge into tunnels. Little research has examined this problem in the past. Using Monte Carlo simulation and information of statistics of heterogeneity, we evaluate the most likely (ensemble average) discharge rate into a tunnel in geologic media with the multi-scale heterogeneity and uncertainty associated with this estimate. We find that the ensemble average discharge rate is larger than the discharge rate predicted by geometric mean of hydraulic conductivity, and smaller than the discharge rate predicted by arithmetic mean of hydraulic conductivity. Moreover, the ensemble average discharge rate decreases with the decay exponent of the depth–decaying trend, and increases with the standard deviation as well as the correlation scale of the stationary log-conductivity fields. The largest uncertainty of discharge rate prediction in the shallow subsurface is controlled by the variability of conductivity fields and the uncertainty at the deep subsurface is by the depth–decaying trend of hydraulic conductivity. Therefore, accurate prediction of groundwater discharge into tunnels requires detailed characterization of multi-scale heterogeneity.

1. Introduction

Groundwater discharge into tunnels could be potential geologic hazards and would affect progress and cost of tunnel excavation. Accurate prediction of the amount and rate of groundwater flow into tunnels is one of the most challenging but essential tasks in geological engineering. If the discharge rate and amount as well as the uncertainty can be determined in advance, proper pumping facilities or suitable drainage systems can be designed to avoid unexpected difficulties as well as costs. Therefore, numerous analytical or semi-analytical equations had been developed to predict the discharge rate per unit length of tunnel (Goodman, 1965; Lei, 1999; El Tani, 2003; Hwang and Lu, 2007; Kolymbas and Wagner, 2007; Park et al., 2008; Arjnoi et al., 2009). In all of the aforementioned equations, hydraulic conductivity is a key parameter. However, the spatial distribution of hydraulic conductivity is complex and difficult to be fully characterized. As a consequence, the geometric or arithmetic mean of sparse hydraulic conductivity measurements from such methods as pumping tests, packer injection tests and slug tests (Chang and Zhang, 2007) are often used to approximate the equivalent homogeneous hydraulic conductivity of the study area and to obtain the estimate of discharge rate. This estimated discharge is likely different from the true one and thus uncertainty associated with the estimate must be quantified (Yeh, 1992).

It is a well-known fact that geologic media exhibit both random (statistically homogeneous) and systematic (e.g., zonation and trending) variability of hydraulic conductivity. Effects of random variability of hydraulic conductivity have been investigated intensively in the last several decades (Gelhar, 1993). Most of these studies however generally ignored effects of large-scale variability in hydraulic conductivity. The widely observed phenomenon of gradual decrease in hydraulic conductivity with depth (Manning and Ingebritsen, 1999; Saar and Manga, 2004; Jiang et al., 2009b; Wang et al., 2009), which is a form of systematic variability of hydraulic conductivity, would greatly control the pattern of groundwater discharge into tunnels at different depths (Zhang and Franklin, 1993). For the problem of predicting groundwater discharge into tunnels, consideration of both random variability...
and depth-decaying trend of hydraulic conductivity would be necessary.

While stochastic methods have been frequently applied to groundwater studies, its applications to studies of groundwater discharge into tunnels are limited. Ando et al. (2003) investigated the tunnel discharge by modeling the rock as a statistically homogeneous stochastic continuum in two dimensions. Up to now, little research has investigated the combined effects of the statistically homogeneous variability of hydraulic conductivity and the widely observed depth-decaying trend on tunnel discharge estimations. The aim of the current study is to fill this gap, which is of both practical and scientific necessity.

2. Methods

In this study, Monte Carlo method, which is conceptually simple and widely applied in stochastic hydrogeology (e.g., Freeze, 1975; Revelli and Ridolfi, 2000), is used to estimate the most likely groundwater discharge into a tunnel based on limited information of the hydraulic conductivity field and quantify uncertainty associated with the estimate at different depths in a formation.

Specifically, we assume that the local-scale heterogeneity of a geological formation can be represented as a stochastic process (a random field), characterized by mean and a covariance function. On the other hand, the regional-scale heterogeneity, i.e., the systematic decay in hydraulic conductivity with depth, can be described using a deterministic trend in the form of an exponential function. We further assume that characterizing the hydraulic conductivity at every point of the formation is impossible. It is however possible to obtain the mean and covariance of the local hydraulic conductivity heterogeneity of the formation as well as its decay trend based on a limited amount of data. As a result, with the known mean, covariance, and trend, many realizations of hydraulic conductivity fields, $K(x, z)$, can be generated which include both local-scale heterogeneity and regional-scale heterogeneity. Subsequently, these conductivity fields can be used in a 2-D numerical steady flow model to calculate all possible groundwater discharges into the tunnel. The average of all possible groundwater discharges (i.e., ensemble average) would then represent the estimate of the most likely groundwater discharge into the tunnel, and the deviation around the average then can be used as a measure of uncertainty associated with the most likely estimate.

2.1. The mathematical model of tunnel discharge

For mathematical simplicity, cross-sectional model is employed in this study. Assume that there is a tunnel of circular cross-section with a diameter of $d$ below sea or lake bed, which has an elevation of $z_{surf}$. By assuming locally isotropic but heterogeneous hydraulic conductivity, the governing equation for two-dimensional steady-state groundwater flow is

$$\frac{\partial}{\partial x} \left( K \frac{\partial H}{\partial x} \right) + \frac{\partial}{\partial z} \left( K \frac{\partial H}{\partial z} \right) = 0$$

(1)

where $K$ is hydraulic conductivity, and $H$ is the hydraulic head at any position. Following Zhang and Franklin (1993), the boundary condition at $z = z_{surf}$, $x = -\infty$ and $x = \infty$ is constant head, which equals $H_0$. To model tunnel discharge, the boundary condition at the tunnel perimeter also needs to be specified. Two different approaches along the tunnel perimeter can be found in the literature. In some studies, zero water pressure is assumed (e.g., El Tani, 2003), while in others, constant total head is used (e.g., Lei, 1999). In this study, we use the former approach.

Analytical solution for tunnel discharge by assuming an exponential decay in hydraulic conductivity with depth can be found in literatures (Zhang and Franklin, 1993; El Tani, 2010). However, it is difficult, if possible, to obtain an analytical solution if hydraulic conductivity shows both random and systematic variability. Many commercial codes can be used for numerical simulation of tunnel discharge, for example, Yang and Wang (2006) used MODFLOW, Arjnoi et al. (2009) used ABAQUS, and Li et al. (2009) used FLAC3D. Here, we use COMSOL Multiphysics, a finite element software, to numerically calculate hydraulic head distribution and amount of tunnel discharge in a cross-section with a domain size of 1 km by 1 km (Fig. 1). The radius of the tunnel with varying depth at the middle of the domain is assumed to be 2 m. Following Zhang and Franklin (1993), the top, bottom, left and right boundaries are all set as constant head boundaries. After obtaining the hydraulic head distribution throughout the domain, the discharge amount is calculated as the normal flux toward the tunnel. The size of meshes around the tunnel is refined to insure accurate results of normal flux.

2.2. The characterization of average trend of decay in hydraulic conductivity with depth

Numerous equations, including theoretical, semi-empirical, and empirical models, relating hydraulic conductivity with depth (stress) can be found in the literature. Jiang et al. (2010) derived semi-empirical equations for depth-decaying hydraulic conductivity and found that the widely accepted empirical exponential decay model can be considered as a special case of their equations under certain simplifications. In this study, therefore, we use the exponential decay model, which can be written as:

$$R(z) = K_{surf} \exp[-A(z_{surf} - z)]$$

(2)

where $R(z)$ is the geometric mean of hydraulic conductivity at points with elevation $z$ in the formation, $K_{surf}$ is the hydraulic conductivity at ground surface, $z_{surf}$ is the elevation of ground surface, and $A$ is the decay exponent, which indicates the rate of decrease in hydraulic conductivity with depth.

Using logarithm transformation, Eq. (2) is equivalent to

$$\ln R(z) = \ln K_{surf} - A(z_{surf} - z)$$

(3)

2.3. Generation of hydraulic conductivity fields with depth-decaying mean and standard deviation

The hydraulic conductivity fields for numerical simulation are generated following the procedures listed below:

(1) A series of statistically homogeneous $\ln K_0(x, z)$ fields with an arithmetic mean of 0, a standard deviation of $\sigma_{\ln K_0}$, and an isotropic covariance function with a correlation scale of

$$\frac{\partial}{\partial x} \left[ K \frac{\partial H}{\partial x} \right] + \frac{\partial}{\partial z} \left[ K \frac{\partial H}{\partial z} \right] = 0$$

$z_{surf}$
Lin both horizontal and vertical directions are generated using SGSIM in GSLib (Deutsch and Journel, 1998). The corresponding $K_0(x,z)$ fields have a geometric mean of $K_{0G} = 1$, an arithmetic mean of $K_{0A} = \exp \left( \frac{1}{2} \sigma_{\ln K_0} \right)$, and a standard deviation of $r_{\ln K_0} = \exp \left( \frac{1}{2} \sigma_{\ln K_0}^2 \right) - 1$, all of which are theoretically independent of depth. The dimension of each grid of the $\ln K_0(x,z)$ fields is 5 m $\times$ 5 m. Note that this is not the same as the dimension of meshes for FEM simulation.

(2) The average trend of depth-decaying hydraulic conductivity after logarithm transformation, $\ln K(z)$ are superimposed onto the statistically homogeneous $\ln K_0(x,z)$ fields. In this way, the resulting $K(x,z)$ fields have a linearly depth-decaying mean, $\ln K(z)$, and a depth-invariant standard deviation, $\sigma_{\ln K_0}$, which equals the standard deviation of $\ln K_0(x,z)$ fields.

(3) The $\ln K(x,z)$ fields are transformed into $K(x,z)$ fields, which are readily available for input into numerical models.

The resulting $K(x,z)$ fields have a geometric mean of $K_g(z) = \exp \left[ \ln K_{surf} - A(z_{surf} - z) \right]$, an arithmetic mean of $K_A(z) = \exp \left[ \ln K_{surf} - A(z_{surf} - z) + \frac{1}{2} \sigma_{\ln K_0}^2 \right]$, and a standard deviation of $\sigma_K(z) = \exp \left[ -A(z_{surf} - z) + \frac{1}{2} \sigma_{\ln K_0}^2 \right] \sqrt{\exp(\sigma_{\ln K_0}^2) - 1}$, all of which are theoretically independent of depth.

Fig. 2. The distribution of hydraulic conductivity, hydraulic head and streamlines towards a circular tunnel. (a) $A = 0$ m$^{-1}$, and no random variability; (b) $A = 0$ m$^{-1}$, $L = 100$ m, and $\sigma_{\ln K_0} = 1$; (c) $A = 0.005$ m$^{-1}$, and no random variability; (d) $A = 0.005$ m$^{-1}$, $L = 100$ m, and $\sigma_{\ln K_0} = 1$.

Fig. 3. The ensemble average discharge rate versus depth for $\ln K_0$ fields ($L = 100$ m and $\sigma_{\ln K_0} = 1$) combined with different rate of depth-decaying trend.
decrease with depth. Note that the coefficient of variation of $K_0(x,z)$ and $K(x,z)$ fields are the same, which equals $\sqrt{\exp(\sigma_{lnK_0}^2) - 1}$.

2.4. Parameters

In this study, $K_{surf}$ is set to be $1.16 \times 10^{-5}$ m/s, which is equivalent to 1 m/d. A series of decay exponent, $A$, ranging between 0 and 0.01 m$^{-1}$ are selected to scrutinize the sensitivity of discharge to decay rate of hydraulic conductivity. More details on the $A$ values of geological media can be found in Jiang et al. (2009a) and Wan et al. (2010). A series of correlation scale, $L$, ranging between 20 and 500 m are chosen to examine the sensitivity of discharge to spatial correlation of hydraulic conductivity. A series of standard deviation, $\sigma_{lnK_0}$, ranging between 0.5 and 2 are selected to identify the sensitivity of discharge to variance of hydraulic conductivity. The tunnel ranges in depth from 20 to 500 m. It is found that when the number of realizations reaches 400, the mean and standard deviation of generated $lnK_0(x,z)$ fields are stable. Moreover, when the number of realizations reaches 400, the ensemble average and variance of discharge rate are also stable. Therefore, we generate 400 realizations for each specified correlation scale.

Typical spatial distribution of hydraulic conductivity under different conditions is illustrated in Fig. 2. Fig. 2a is a purely homogeneous hydraulic conductivity field, while Fig. 2b shows a statistically homogeneous hydraulic conductivity field with a correlation scale of $L = 100$ m and a standard deviation of $\sigma_{lnK_0} = 1$. When the decay exponent of hydraulic conductivity equals 0.005 m$^{-1}$, Fig. 2a is transformed to Fig. 2c, and Fig. 2b is trans-
formed to Fig. 2d. Note that in Fig. 2b, the maximum hydraulic conductivity is about three orders of magnitude larger than the minimum hydraulic conductivity, while in Fig. 2c, the hydraulic conductivity in the bottom of the domain and in the surface of the domain differs greatly with a ratio of more than two orders of magnitude. By combining the depth–decaying trend and random variability of hydraulic conductivity, \( \log_{10} K \) of the hydraulic conductivity field shown in Fig. 2d ranges between \(-8.1\) and \(-3.9\).

3. The ensemble average discharge rate versus depth

In this section, discharge rate at different depths is calculated for each of the 400 realizations under some given conditions, and then the ensemble average (i.e., the most likely) discharge rate at each depth is determined and compared with the discharge rate correspondent to hydraulic conductivity fields with the geometric and arithmetic means of the 400 realizations at a given depth (i.e., \( K_c(z) \) and \( K_0(z) \)), both of which are dependent on the decay exponent.

3.1. The influence of decay exponent on discharge–depth pattern

Zhang and Franklin (1993) discussed the effect of decay exponent on the discharge rate at different depths under depth–decaying hydraulic conductivity without random variability, which is equivalent to the discharge rate corresponding to a hydraulic conductivity with \( K_c(z) \) in this study. Here, we examine the sensitivity of discharge rate to decay exponent by comparing the ensemble average discharge rate from 400 realizations, \( q_{\text{mean}} \), the discharge rate correspondent to \( K_c(z) \), \( q_c \), and the discharge rate correspondent to \( K_0(z) \), \( q_0 \). All of which are depth–dependent. The ln \( K_0(x, z) \) fields have a correlation scale of \( r = 100 \) m and a standard deviation of \( \sigma_{\ln K_0} = 1 \).

In Fig. 2, it is shown that under depth–decaying hydraulic conductivity, the number of streamlines decreases, which in turn reduces the discharge rate. The pattern of ensemble average discharge rate versus depth is shown in Fig. 3, which is similar to the results given in Zhang and Franklin (1993). Fig. 4 shows the difference between ensemble average discharge rate and discharge rates \( q_0 \) and \( q_c \) as a function of depth. The pattern of \( q_0 \) versus depth is similar to that of \( q_c \) versus depth, with a ratio of \( q_0/q_c \), which equals to \( \exp[2 \sigma_{\ln K_0}^2] \). For all cases shown in Fig. 3, the curves for \( q_{\text{mean}} \)-depth are bounded by the two curves for \( q_0 \) and \( q_c \) with similar patterns.

3.2. The influences of \( \sigma_{\ln K_0} \) and correlation scale on discharge–depth pattern

The three parameters, i.e., mean, standard deviation and correlation scale (correspondent to range in geostatistics) are the key to characterize stochastic hydraulic conductivity fields. As discussed in Section 2.3, the mean of the hydraulic conductivity fields with both random variability and depth–decaying trend is mainly dependent on the decay exponent, \( A \), if \( K_{\text{surf}} \) is fixed. In the following section, we will keep the decay exponent as a constant \( A = 0.005 \) m\(^{-1} \) and discuss the influence of random variability on the pattern of \( q_{\text{mean}} \) versus depth by comparing ln \( K_0 \) fields with different correlation scales and different standard deviations (Fig. 5).

When the correlation scale is kept to be 100 m and \( \sigma_{\ln K_0} \) ranges between 0.5 and 2, all the curves of \( q_{\text{mean}} \)-depth are to the right of the curve of \( q_c \)-depth (Fig. 5a). Moreover, the deviation of the \( q_{\text{mean}} \)-depth curve from the \( q_c \)-depth curve increases with \( \sigma_{\ln K_0} \), which implies that the discharge rate increases with the standard deviation of ln \( K_0 \). Since \( K_0 \) is also dependent on \( \sigma_{\ln K_0} \), the curves for \( q_A \) are not shown in Fig. 5a. Note that when \( \sigma_{\ln K_0} \) exceeds 1.414, the pattern of \( q_{\text{mean}} \) versus depth is no longer similar to that of \( q_c \) versus depth. This might be a result of preferential paths due to the large variance of hydraulic conductivity.

Fig. 5b shows the curves of \( q_{\text{mean}} \)-depth when \( \sigma_{\ln K_0} \) is kept to be 1 and the correlation scale increases from 50 m to 200 m. Also shown are the curves for \( q_c \) and \( q_A \). All the curves of \( q_{\text{mean}} \) versus depth lie between the curves of \( q_c \) and \( q_A \) versus depth, and as the correlation scale increases, the \( q_{\text{mean}} \) increases towards \( q_A \).

4. The ensemble average discharge rate of specific tunnels

The influence of depth–decaying trend and random variability of hydraulic conductivity on discharge–depth pattern is shown in Fig. 6. The variation in ensemble average discharge rate with decay exponent, standard deviation and correlation scale in tunnels at three different depths. (a) Discharge rate versus decay exponent, \( A \); (b) discharge rate versus standard deviation, \( \sigma_{\ln K_0} \); (c) discharge rate versus correlation scale, \( L \).
Section 3. Below, we discuss the influencing factors of discharge rate of a tunnel at specific depths.

4.1. The influence of decay exponent, $\sigma_{\ln K_0}$ and correlation scale on discharge rate of a tunnel

We choose three tunnels, which are 80 m, 160 m and 240 m below surface, to discuss the influences of decay exponent, and standard deviation and correlation scale of log-conductivity fields on discharge rate. When the correlation scale is kept to be 100 m, $\sigma_{\ln K_0}$ is kept to be 1, $q_{\text{mean}}$ decreases nonlinearly with the decay exponent (Fig. 6a). A similar phenomenon has been observed by Zhang and Franklin (1993), which reports that for the cases of depth–decaying hydraulic conductivity, the discharge rate decreases with the decay exponent. Moreover, as the depth of the tunnel increases, the rate of decrease in discharge rate with decay exponent increases.

When the decay exponent is kept to be 0.005 m$^{-1}$, and correlation scale is kept to be 100 m, $q_{\text{mean}}$ increases nonlinearly with $\sigma_{\ln K_0}$ (Fig. 6b). When the decay exponent is kept to be 0.005 m$^{-1}$, $\sigma_{\ln K_0}$ is kept to be 1, $q_{\text{mean}}$ also increases nonlinearly with correlation scale (Fig. 6c). However, the $q_{\text{mean}} - \sigma_{\ln K_0}$ curves convex downward while $q_{\text{mean}} - L$ curves convex upward. This result is logical: under a constant rate of depth–decaying trend as in this case, the discharge increases with the variability of hydraulic conductivity, representing an increase in preferential flow paths. On the other hand, as the correlation scale increases, the medium approaches homogeneity and the variance reflects only the uncertainty about the homogeneous properties, in stead of the spatial variability.

4.2. The influence of local average hydraulic conductivity on discharge rate of a tunnel

It is well-known that the discharge rate of a well is significantly influenced by the hydraulic conductivity in the immediate vicinity of the well. Previous studies reported that near the well the effective hydraulic conductivity is the arithmetic mean of hydraulic conductivity if the well is considered as constant head (Indelman and Dagan, 2004). Therefore, we use the $K_a$ of cells around the tunnel to establish the relationship between discharge rate of a tunnel and local average hydraulic conductivity. When $A$ is fixed, for each of the 400 ln $K_0$ realizations with a certain correlation scale, $L$, the arithmetic mean of $K_0$ in windows surrounding a tunnel with different sizes are calculated. In this way, for each correlation scale and each window size, 400 sets of $K_a$ versus discharge rate can be obtained and can be used to calculate the coefficient of correlation, $R$, between $K_a$ and the discharge rate. Then $R$ versus window size for each correlation scale can be plotted.

For the tunnel with a depth of 500 m, the plot of $R$ versus window size for different correlation scales when $A = 0$ is shown in Fig. 7. It is clear that the size of window with maximum $R$ increases with the correlation scale of the hydraulic conductivity field. When the correlation scale equals 10 m, the size of window with maximum $R$ is only $4 \times 4$ cells, corresponding to 20 m $\times$ 20 m. When the correlation scale reaches 500 m, the size of window with maximum $R$ is as large as $18 \times 18$ cells, corresponding to 90 m $\times$ 90 m. This is a direct result of the continuity of hydraulic conductivity.

![Fig. 7. The coefficient of correlation, $R$, between $K_a$ and the average discharge rate versus window size for different correlation scales for the tunnel 500 m below surface when $A = 0$.](image)

![Fig. 8. The depth–dependent coefficient of variation of discharge rate under different conditions. (a) For different $A$ when $L = 100$ m and $\sigma_{\ln K_0} = 1$; (b) for different $\sigma_{\ln K_0}$ when $L = 100$ m and $A = 0.005$ m$^{-1}$; (c) for different $L$ when $\sigma_{\ln K_0} = 1$ and $A = 0.005$ m$^{-1}$.](image)
distribution. A hydraulic conductivity field with a large correlation scale implies a good continuity.

5. The uncertainty of discharge rate versus depth

Variance or standard deviation of the tunnel discharge can be used as a measure of likely deviation of predicted discharge using the corrected mean from the reality (i.e., uncertainty in prediction). However, when the mean differs greatly as in our study shown in Section 3, we prefer to use coefficient of variation, CV, to characterize uncertainty. Here, we compare the coefficient of variation under different cases, which are found to be depth–dependent (Fig. 8).

When four hundred realizations of initial ln $K_0$ fields with a correlation scale of 100 m and a standard deviation of 1 are combined with a series of decay exponents (five different decay exponents in this study), the CV of discharge rate for each decay exponent mainly decreases with depth but increases with the decay exponent (Fig. 8a).

When five sets of four hundred realizations of initial ln $K_0$ fields with the same correlation scale but different standard deviations are added by a depth–decaying trend with a decay exponent of 0.005 m$^{-1}$, the CV of discharge rate for each standard deviation of ln $K_0$ mainly decreases with depth but increases with the standard deviation (Fig. 8b).

When five sets of four hundred realizations of initial ln $K_0$ fields with a standard deviation of 1 but different correlation scale are added by a depth–decaying trend with a decay exponent of 0.005 m$^{-1}$, the CV of discharge rate for each correlation scale of ln $K_0$ mainly decreases with depth but increases with the correlation scale (Fig. 8c).

Fig. 9 shows the coefficient of variation of discharge rate due to uncertainty in decay exponent, correlation scale and standard deviation of ln $K_0$. If the decay exponent is unknown, then the CV of discharge rate calculated from the 2000 realizations (5 x 400 realizations) decreases with depth in the shallow part first and then increases with depth when the tunnel is 100 m below surface. On the other hand, if the standard deviation of ln $K_0$ is unknown, then the CV of discharge rate calculated from the 2000 realizations generally decreases with depth. If the correlation scale of ln $K_0$ is unknown, the CV of discharge rate calculated from 2000 realizations also decreases with depth. At all depths as shown in Fig. 9, the uncertainty of discharge rate caused by uncertainty in correlation scale of ln $K_0$ is the lowest. In the shallow part (above 220 m in this study), the uncertainty of discharge rate caused by uncertainty in ln $K_0$ is the largest, while in the deep part (below 220 m in this study), the uncertainty of discharge rate caused by uncertainty in the rate of depth–decaying trend is the largest.

6. Concluding remarks

In this paper, Monte Carlo method is used to simulate groundwater discharge into tunnels in formations with both random and systematic variability of hydraulic conductivity. The systematic variability is represented as exponential decay in hydraulic conductivity with depth and the random variability is represented by a stochastic process. As a result, the mean of hydraulic conductivity is mainly dependent on varying decay exponent and the random variability of hydraulic conductivity is described by varying standard deviation and correlation scale of log-conductivity fields. Hydraulic conductivity fields with different decay exponents, standard deviations, and correlation scales are generated and used as input into steady-state groundwater flow models to simulate tunnel discharge rates and investigate uncertainty associated with predictions.

Since exact discharge rate for a given tunnel is difficult to predict, hydrogeologist and geotechnical engineer often resort to predict the most likely discharge rate under uncertainty. This most likely discharge rate is the ensemble average discharge rate, which is found to decrease with the decay exponent, but increases with the standard deviation and correlation scale of log-conductivity fields. The curves for ensemble average discharge rate versus depth, whose shape is highly dependent on the decay exponent, lies between the curves of discharge rate versus depth obtained from geometric mean and arithmetic mean of hydraulic conductivity. If the mean of hydraulic conductivity is used to predict the most likely discharge rate, geometric mean would lead to a too conservative result. On the contrary, arithmetic mean would lead to a too large discharge rate. The discharge rate of a tunnel is also highly dependent on the hydraulic conductivity near the tunnel. The size of influencing region is dependent on the correlation scale of log-conductivity fields. When the correlation scale is large, which implies a high continuity of hydraulic conductivity, the size of influencing region is also large. Lastly, we show that uncertainty in discharge rate (the deviation of the ensemble mean discharge rate from one particular realization) can be very significant and is mainly controlled by the standard deviation of log-conductivity fields in the shallow subsurface but controlled by the decay exponent of hydraulic conductivity in the deep subsurface. Therefore, detailed information of both local-scale heterogeneity and regional-scale heterogeneity is indispensable for accurate prediction of groundwater discharge into tunnels. However, when the tunnel is deep, decay exponent is the most critical parameter to ensure the accuracy of tunnel discharge prediction.

Emerging new technology such as hydraulic tomography (Yeh and Liu, 2000; Zhu and Yeh, 2005; Illman et al., 2009; Xiang et al., 2009) could be a useful tool for improving our characterization of hydraulic properties distribution and in turn accuracy of our prediction of discharge into tunnels in the subsurface.

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